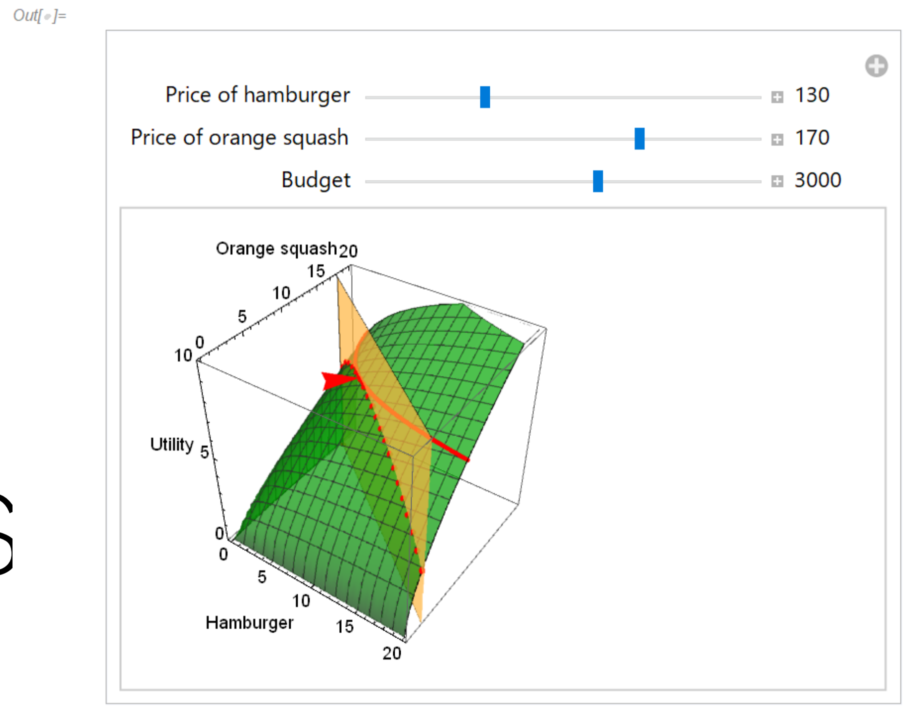


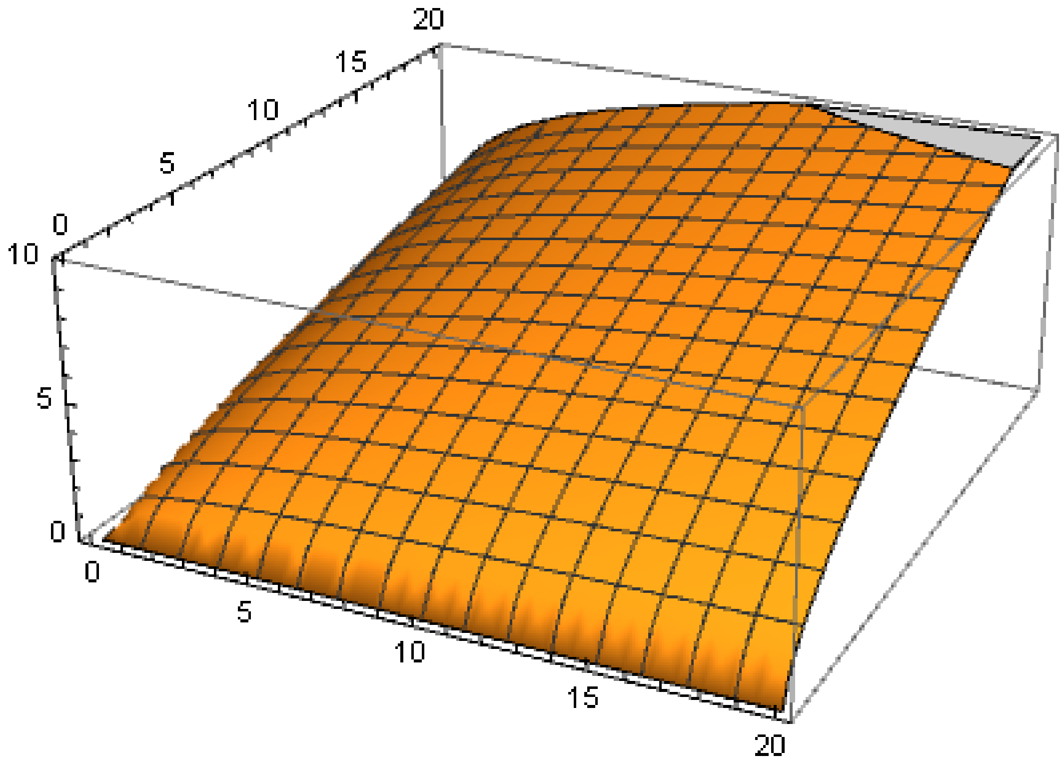
Let's Make the 3D Graphics

18th May, 2019

Gakushuin University, Faculty of Economics
Prof. Yukari SHIROTA

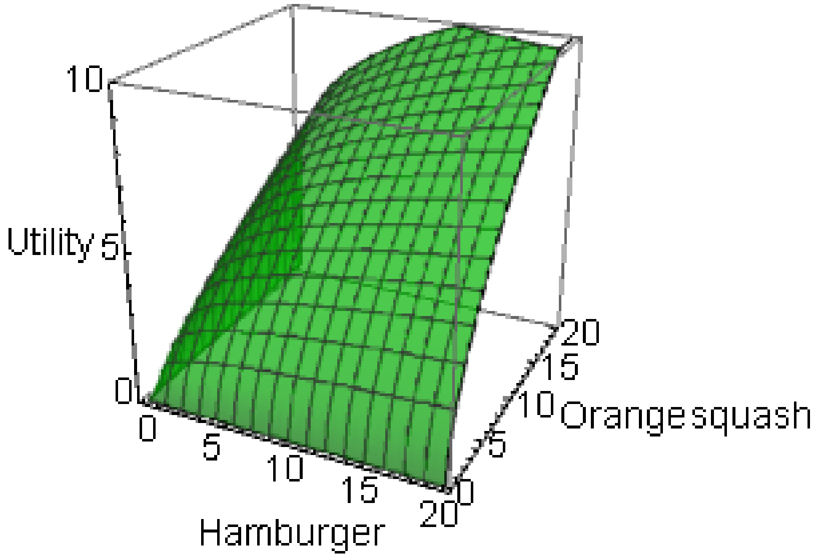
3D Plot

Plot3D[x^0.2 * y^0.6, {x, 0, 20}, {y, 0, 20}, PlotRange -> {0, 10}]

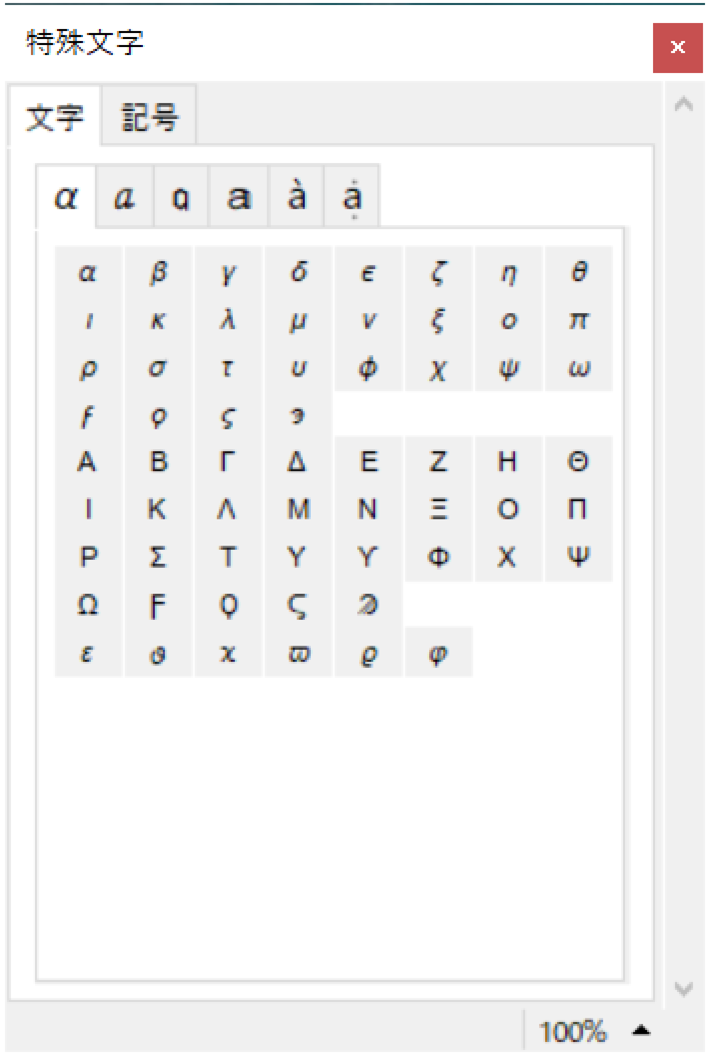


3D Plot with other options

```
Plot3D[ x^0.2 * y^0.6, {x, 0, 20}, {y, 0, 20}, PlotRange -> {0, 10},
 BoxRatios -> 1, PlotStyle -> {Opacity[0.7], Green},
 AxesLabel -> {"Hamburger", "Orange squash", "Utility"},
 ImageSize -> 200]
```



Pallet for special characters from the menu



Conduct the Lagrange method.

Definition of Lagrange function:

```
F[x_,y_,λ_]:=x^0.2 * y^0.6+ λ (M-(P1 x+P2 y));
```

In[ ]:= F[x, y, λ]

Out[ ]= $x^{0.2}\,y^{0.6} + (M - P1\,x - P2\,y)\,\lambda$

Partial differentiation

```
In[ ]:= D[F[x, y, λ],  x]
         微分係数
```

$$\text{Out[ ]=}\ \frac{0.2\,y^{0.6}}{x^{0.8}} - P1\,\lambda$$

- Set of equations

```
In[ ]:= {D[F[x, y, λ], x] == 0, D[F[x, y, λ], y] == 0, M - (P1 x + P2 y) == 0}
         微分係数                 微分係数
```

$$\text{Out[ ]=}\ \left\{\frac{0.2\,y^{0.6}}{x^{0.8}} - P1\,\lambda == 0,\ \frac{0.6\,x^{0.2}}{y^{0.4}} - P2\,\lambda == 0,\ M - P1\,x - P2\,y == 0\right\}$$

Conduct the Lagrange method.

```
In[ ]:= Solve[{D[F[x, y, λ], x] == 0, D[F[x, y, λ], y] == 0, M - (P1 x + P2 y) == 0}, {x, y, λ}]
```
解く　微分係数　微分係数

We can get the 5 answers. We will use the first real number one.

Out[ ]=
$$\left\{\left\{x \to \frac{0.25\,M}{P1},\ y \to \frac{0.75\,M}{P2},\ \lambda \to \frac{0.51017}{M^{1/5}\,P1^{1/5}\,P2^{3/5}}\right\},\right.$$
$$\left\{x \to \frac{(0.25 + 0.\,i)\,M}{P1},\ y \to \frac{(0.75 + 0.\,i)\,M}{P2},\ \lambda \to -\frac{0.412736 + 0.29987\,i}{M^{1/5}\,P1^{1/5}\,P2^{3/5}}\right\},$$
$$\left\{x \to \frac{(0.25 + 0.\,i)\,M}{P1},\ y \to \frac{(0.75 + 0.\,i)\,M}{P2},\ \lambda \to -\frac{0.412736 - 0.29987\,i}{M^{1/5}\,P1^{1/5}\,P2^{3/5}}\right\},$$
$$\left\{x \to \frac{(0.25 + 0.\,i)\,M}{P1},\ y \to \frac{(0.75 + 0.\,i)\,M}{P2},\ \lambda \to \frac{0.157651 - 0.4852\,i}{M^{1/5}\,P1^{1/5}\,P2^{3/5}}\right\},$$
$$\left.\left\{x \to \frac{(0.25 + 0.\,i)\,M}{P1},\ y \to \frac{(0.75 + 0.\,i)\,M}{P2},\ \lambda \to \frac{0.157651 + 0.4852\,i}{M^{1/5}\,P1^{1/5}\,P2^{3/5}}\right\}\right\}$$

Select the first answer by First command

```
In[ ]:= First[Solve[
        最初    解く
          {D[F[x, y, λ], x] == 0, D[F[x, y, λ], y] == 0, M - (P1 x + P2 y) == 0}, {x, y, λ}]]
           微分係数                 微分係数
```

Out[ ]= $\left\{x \to \frac{0.25\,M}{P1},\ y \to \frac{0.75\,M}{P2},\ \lambda \to \frac{0.51017}{M^{1/5}\,P1^{1/5}\,P2^{3/5}}\right\}$

Replace {x,y,u} by the first answer

Replace a variable with a value:
/.

```
In[ ]:= {x, y, x^0.2 * y^0.6} /. First[Solve[
        最初      解く
   {D[F[x, y, λ], x] == 0, D[F[x, y, λ], y] == 0, M - (P1 x + P2 y) == 0}, {x, y, λ}] ]
    微分係数                  微分係数
```

$$\text{Out[ ]=}\ \left\{\frac{0.25\,M}{P1}, \frac{0.75\,M}{P2}, 0.637712\left(\frac{M}{P1}\right)^{0.2}\left(\frac{M}{P2}\right)^{0.6}\right\}$$

Then pts definition

$$\text{pts} := \left\{\frac{0.25M}{\text{P1}}, \frac{0.75M}{\text{P2}}, 0.637712\left(\frac{M}{\text{P1}}\right)^{0.2}\left(\frac{M}{\text{P2}}\right)^{0.6}\right\}$$

We will use the pts definition repeatedly.

Calculate intersection.

```
In[ ]:= Solve[M - (P1 x + P2 y) == 0, {y}]
解く
```

$$Out[\ ]= \left\{\left\{y \to \frac{M - P1\,x}{P2}\right\}\right\}$$

M=2900; P1=130; P2=170;

zvals=Table[{xx, **(M/P2-xx*P1/P2)**, $xx^{0.2}$ * $(M/P2\text{-}xx*P1/P2)^{0.6}$}, {xx,0,20}]

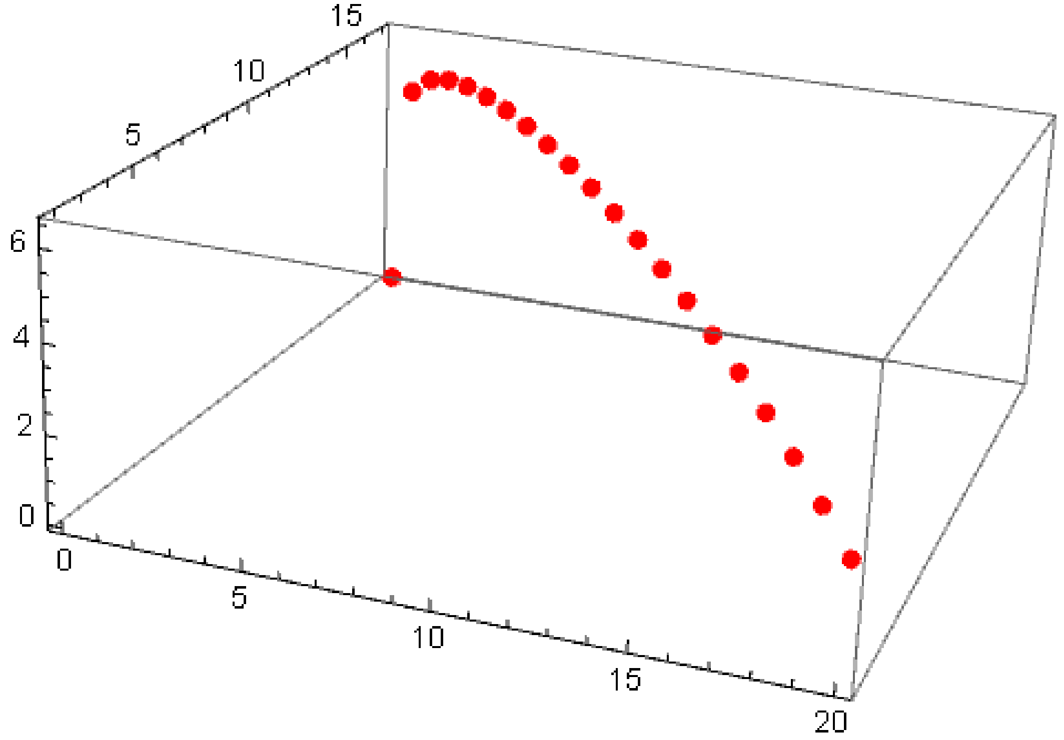
$\{\{0, \frac{290}{17}, 0.\}, \{1, \frac{277}{17}, 5.33603\}, \{2, \frac{264}{17}, 5.95524\}, \{3, \frac{251}{17}, 6.26555\},$
$\{4, 14, 6.42819\}, \{5, \frac{225}{17}, 6.49881\}, \{6, \frac{212}{17}, 6.50373\}, \{7, \frac{199}{17}, 6.45747\},$
$\{8, \frac{186}{17}, 6.36878\}, \{9, \frac{173}{17}, 6.24319\}, \{10, \frac{160}{17}, 6.08419\},$
$\{11, \frac{147}{17}, 5.89386\}, \{12, \frac{134}{17}, 5.67322\}, \{13, \frac{121}{17}, 5.42239\},$
$\{14, \frac{108}{17}, 5.14056\}, \{15, \frac{95}{17}, 4.82595\}, \{16, \frac{82}{17}, 4.47551\},$
$\{17, \frac{69}{17}, 4.08442\}, \{18, \frac{56}{17}, 3.645\}, \{19, \frac{43}{17}, 3.14458\}, \{20, \frac{30}{17}, 2.55982\}\}$

This is the result.
21 points

Let's draw the points.

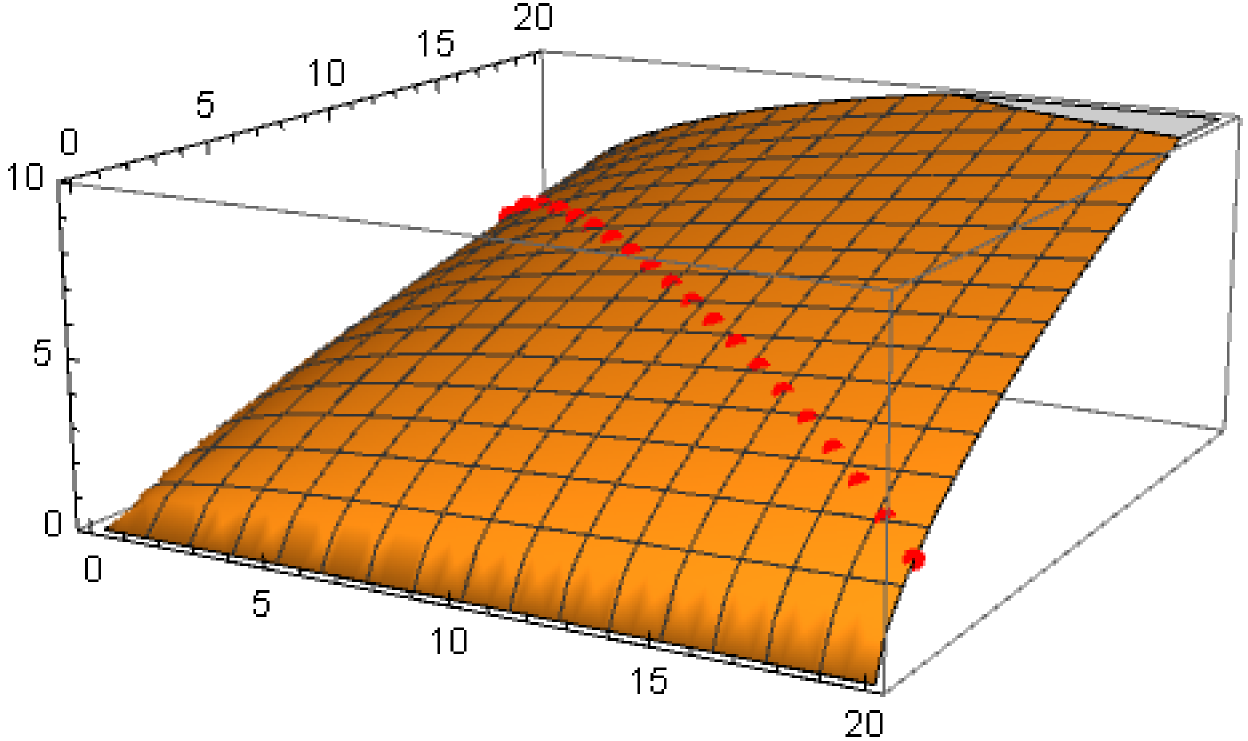
Draw the 21 points.

```
M=2900;P1=130;P2=170;
zvals=Table[{xx,(M/P2-xx*P1/P2),xx^0.2*(M/P2-xx*P1/P2)^0.6}, {xx,0,20}]
ListPointPlot3D[zvals, PlotStyle->Red]
```



```
In[ ]:= Show[Plot3D[x^0.2 * y^0.6, {x, 0, 20}, {y, 0, 20}, PlotRange → {0, 10}],
        示す     3Dプロット                                    プロット範囲
  M = 2900; P1 = 130; P2 = 170;
  zvals =
   Table[{xx, (M / P2 - xx * P1 / P2), xx^0.2 * (M / P2 - xx * P1 / P2)^0.6}, {xx, 0, 20}];
   リストを作成
  ListPointPlot3D[zvals, PlotStyle → Red]]
  3D散布図                 プロットスタイル  赤
```

Out[ ]=



Show
Display several items at the same time.

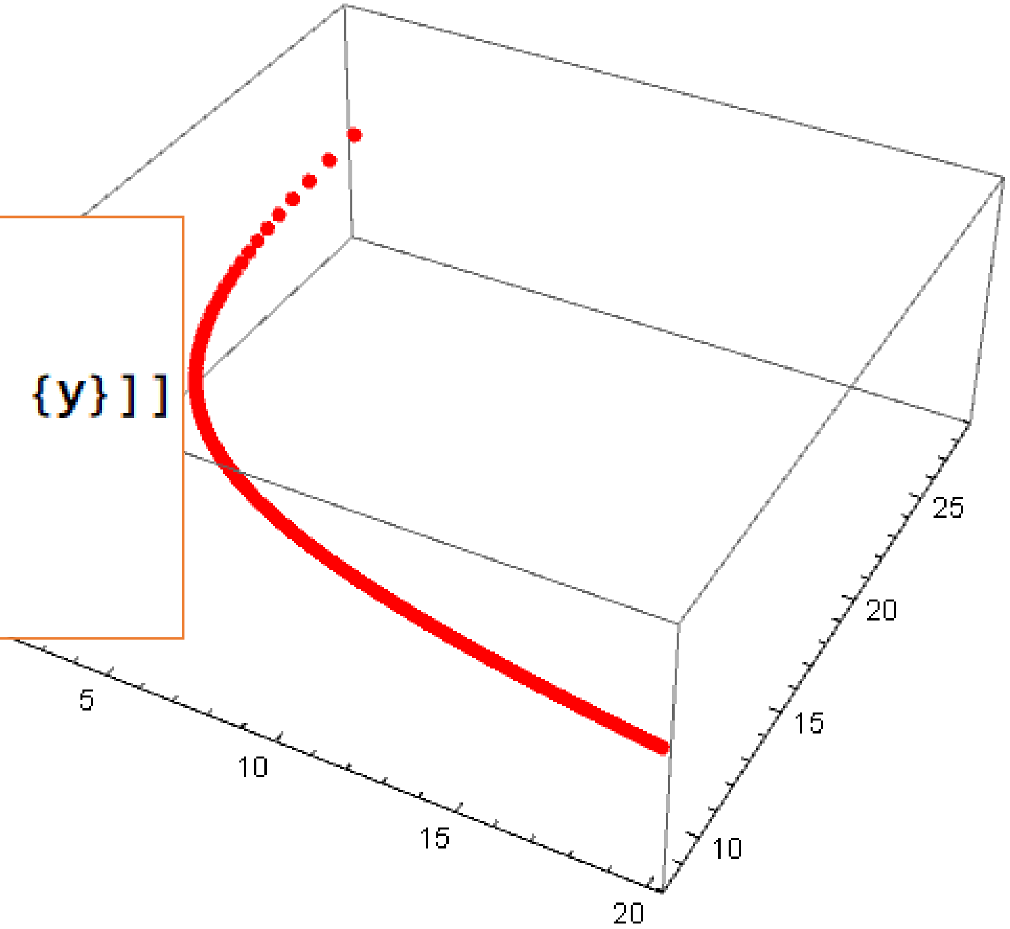
Show[
AAAAAA,
BBBBBB,
CCCCCC]

U's contour

- Given x and U
- Unknown y

From 0.5 to 20 by a step 0.1

```
In[・]:= uval = pts[[3]];
ucurve = Table[{x, uval^(5/3)/x^(1/3), uval}, {x, 0.5, 20, 0.1}];
         リストを作成
ListPointPlot3D[ucurve, PlotStyle → Red]
3D散布図                プロットスタイル  赤
```




```
In[・]:= Clear[uval];
         クリア
Simplify[Solve[uval == x^(1/5)*y^(3/5), {y}]]
簡単な…   解く
```

Out[・]= $\left\{\left\{y \to \left(\frac{uval}{x^{1/5}}\right)^{5/3}\right\}\right\}$

$$y = \frac{uval^{\frac{5}{3}}}{x^{\frac{1}{3}}}$$

U’s contour

- Maximum point

pts[[3]] is

$$\left\{\frac{0.25\,M}{P1},\ \frac{0.75\,M}{P2},\ \boxed{0.637712\left(\frac{M}{P1}\right)^{0.2}\left(\frac{M}{P2}\right)^{0.6}}\right\}$$

Find the u’s contour curve

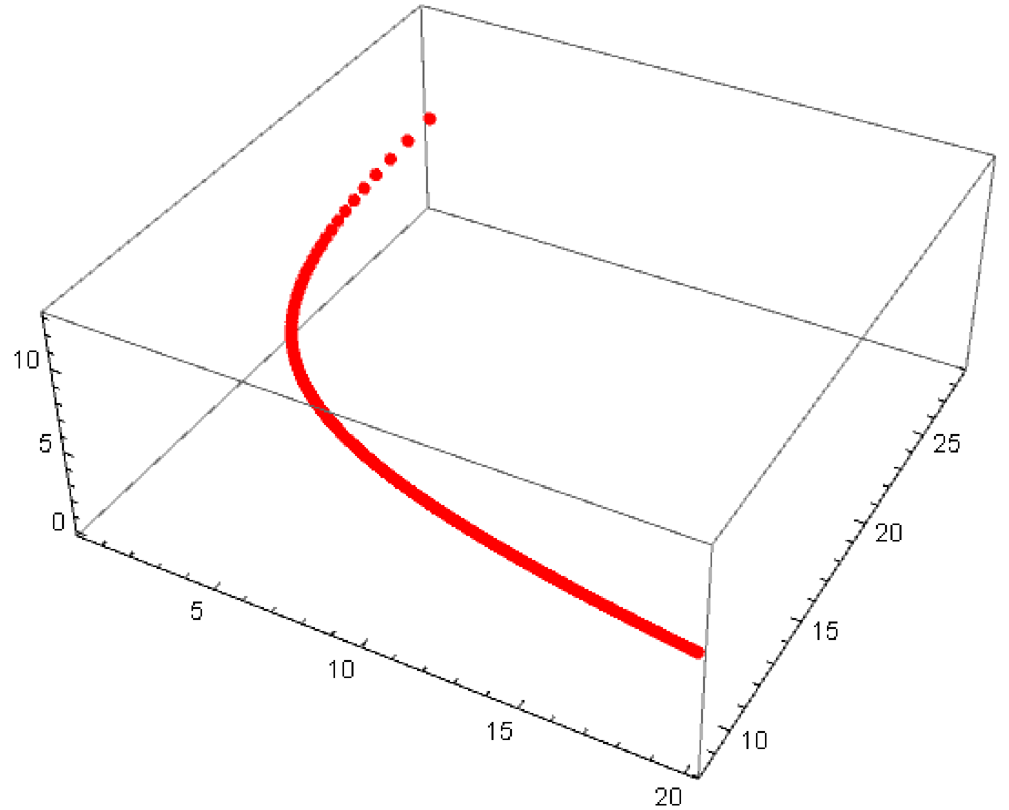
From 0.5 to 20 by a step 0.1

```
In[ ]:= uval = pts[[3]];

        ucurve = Table[{x, uval^(5/3)/x^(1/3), uval}, {x, 0.5, 20, 0.1}];
                 リストを作成

        ListPointPlot3D[ucurve, PlotStyle → Red]
        3D散布図                 プロットスタイル  赤
```

Out[ ]=



LagrangeFEBtext.cdf * - Wolfram Mathematica 11.3

ファイル (F)　編集 (E)　挿入 (I)　書式 (R)　セル (C)　グラフィックス (G)　評価 (V)　パレット (P)　ウィンドウ (W)　ヘルプ (H)

Input

In[47]:= ucurve

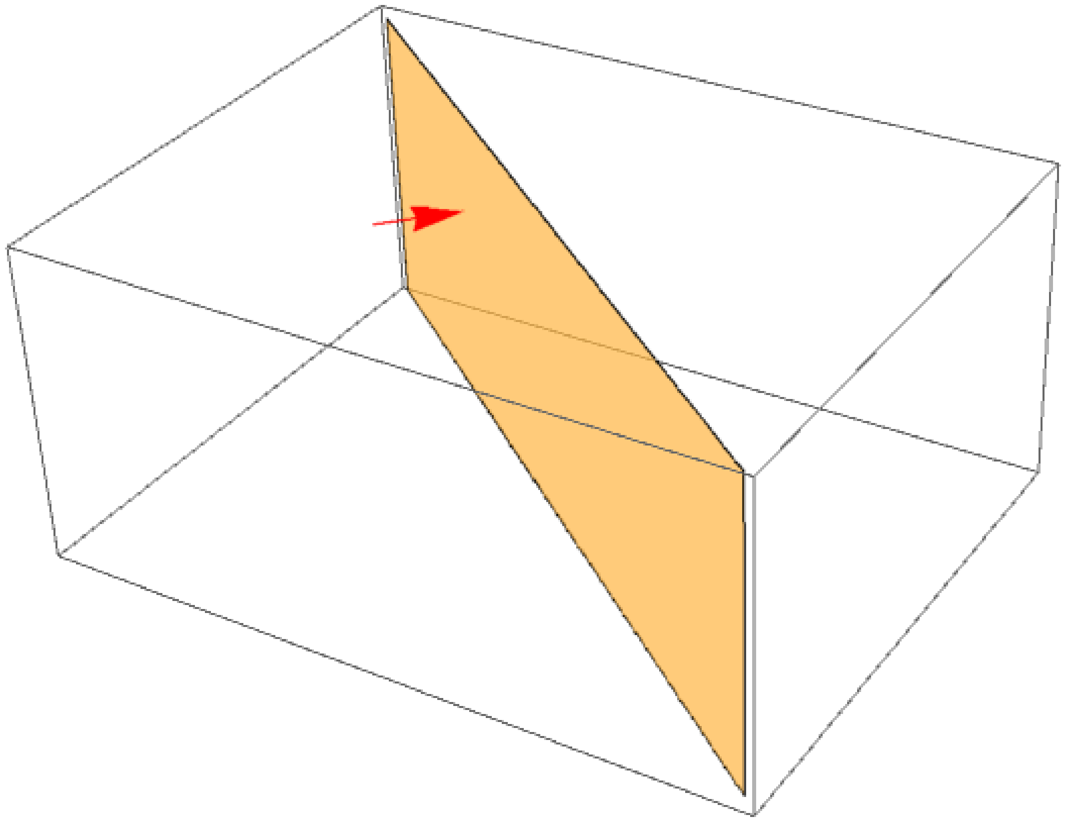
Out[47]= $\left\{\left\{0.5,\ 117.268\left(\left(\frac{P1}{3.92308\,P1+1.\,P2}\right)^{0.6}\left(\frac{P2}{51.\,P1+13.\,P2}\right)^{0.2}\right.\right.\right.$

$15.181\left(\frac{P1}{3.92308\,P1+1.\,P2}\right)^{0.6}\left(\frac{P2}{51.\,P1+13.\,P2}\right)^{0.2}\Big\},$

$\Big\{0.6,\ 110.353\left(\left(\frac{P1}{3.92308\,P1+1.\,P2}\right)^{0.6}\left(\frac{P2}{51.\,P1+13.\,P2}\right)^{0.2}\right)^{5/3},$

Budget Restriction Plane & Maximum point noted by Arrow

Graphics3D[{Polygon[{{0,M/P2,0},{M/P1,0,0},{M/P1,0,10},{0,M/P2,10},{0,M/P2,0}}],
Arrow[{pts-{2,2,0}, pts}]
]



2 graphics items

(1) Polygon

(2) Arrow

The arrow vector is set to be {2,2,0}.

Plane & Arrow

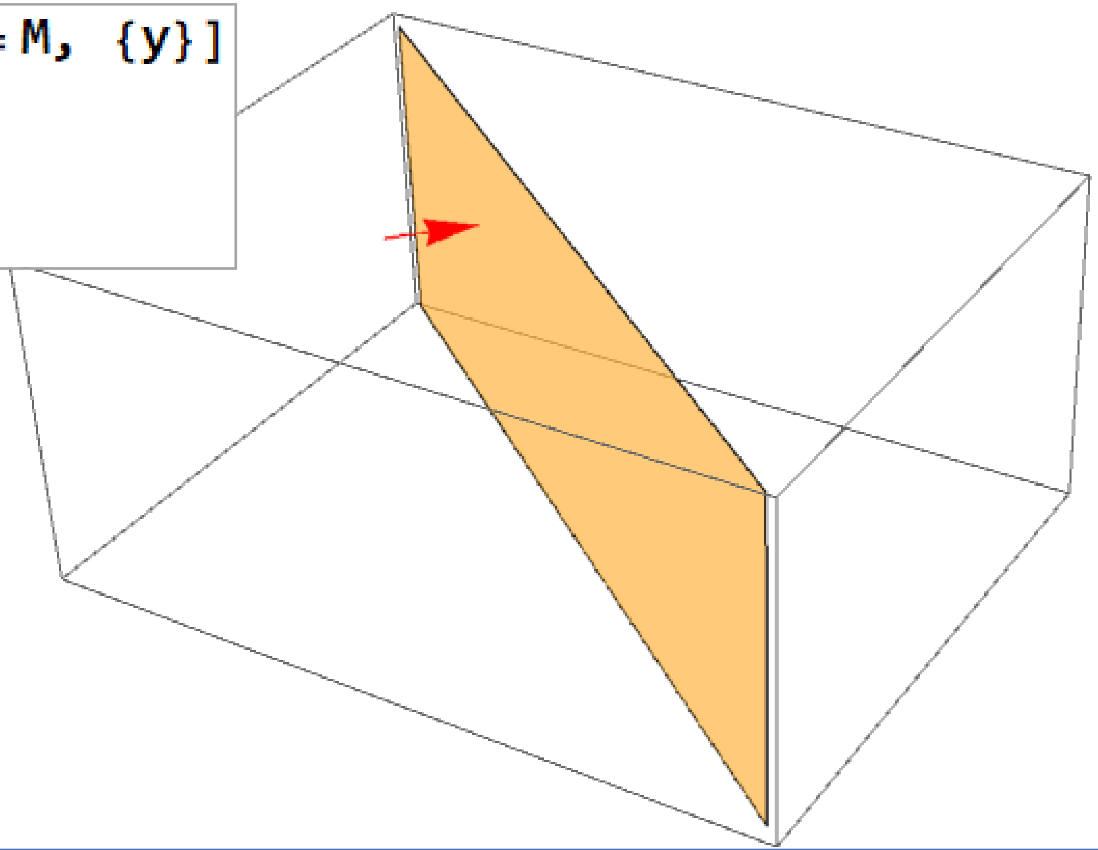
In[▪]:=
```
Show[Graphics3D[{Opacity[0.7], Glow[Orange],
```
示す 3Dグラフィックス 不透明度 光沢 オレンジ色

$$\text{Polygon}\left[\left\{\left\{0, \frac{M}{P2}, 0\right\}, \left\{\frac{M}{P1}, 0, 0\right\}, \left\{\frac{M}{P1}, 0, 10\right\}, \left\{0, \frac{M}{P2}, 10\right\}, \left\{0, \frac{M}{P2}, 0\right\}\right\}\right],$$

多角形

```
Opacity[1], Red, Arrowheads[Large], Arrow[{pts - {2, 2, 0}, pts}]}]]
```
不透明度 赤 鏃 大きい 矢印

Out[▪]=

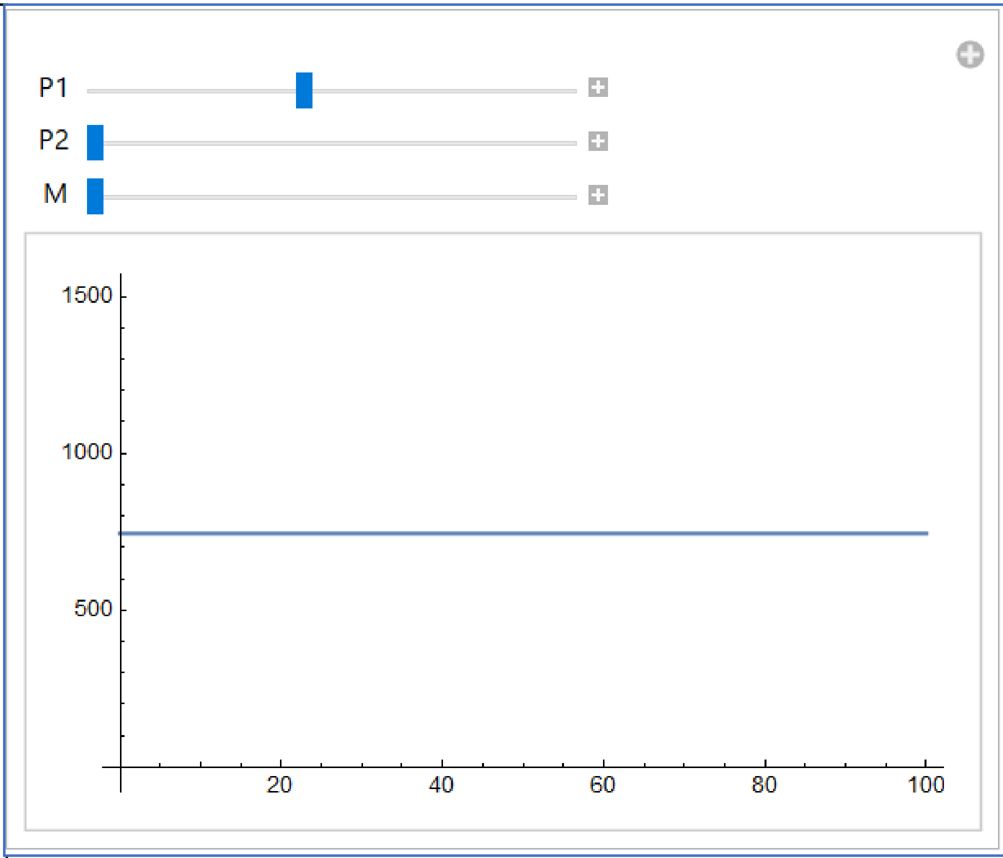


In[▪]:=
```
Solve[0*P1 + y*P2 == M, {y}]
```
解く

Out[▪]= $\left\{\left\{y \to \frac{M}{P2}\right\}\right\}$

Manipulate small test

```
In[ ]:= Manipulate[
        操作
         Show[
         示す
          Plot[P1 + P2 + M, {x, 0, 100}, PlotRange -> All]
          プロット                        プロット範囲    すべて
         ],
         {P1, 100, 200},
         {P2, 100, 200},
         {M, 500, 2100}, SaveDefinitions → True,
                         定義保存            真
         Initialization → {ClearAll[M, P1, P2, F]}
                           すべてをクリア
        ]
```



Change M, P1 and P2 by using sliders
Manipulate

```
Manipulate[
操作
```

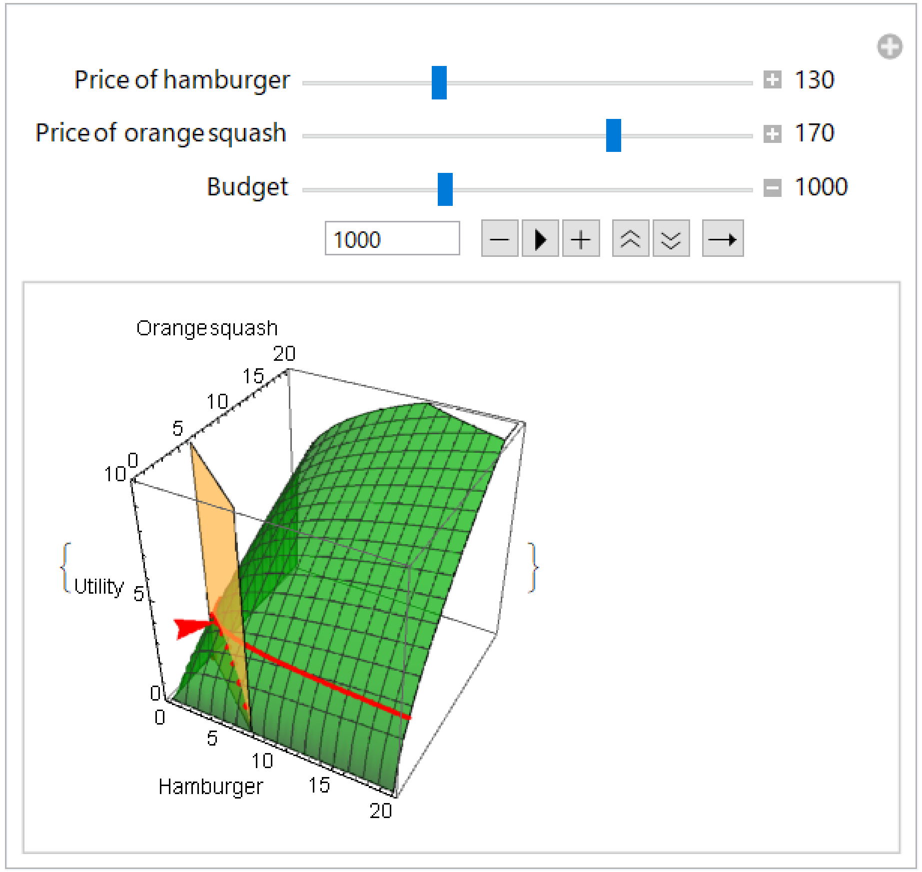
Repeated part

```
{{P1, 130, "Price of hamburger"}, 100, 200, 10, Appearance→ "Labeled"},
                                               外観          ラベル付き
{{P2, 170, "Price of orange squash"}, 100, 200, 10, Appearance→ "Labeled"},
                                                   外観          ラベル付き
{{M, 3000, "Budget"}, 100, 5000, 100, Appearance→ "Labeled"}, SaveDefinitions→ True,
                                      外観          ラベル付き     定義保存          真
```

M with the initial value 3000 of which range is from 100 to 5000 by step 100

Final combine of all parts

- Remove M=2900; P1=130; P2=170; so that the Manipulator can change the values



Drill

- Change the target function to $\sqrt{x} + \sqrt{y}$
- When you are making the drill program, please close the other manipulation program, because in Mathematica variables may be affected by other program variables.