

Effect of the treatment between 2 groups  
to increase the weight (averageA – averageB)  
Data: (The distributions are not normal distributions)  
Test: Mann-Whitney U test (non parametric test)

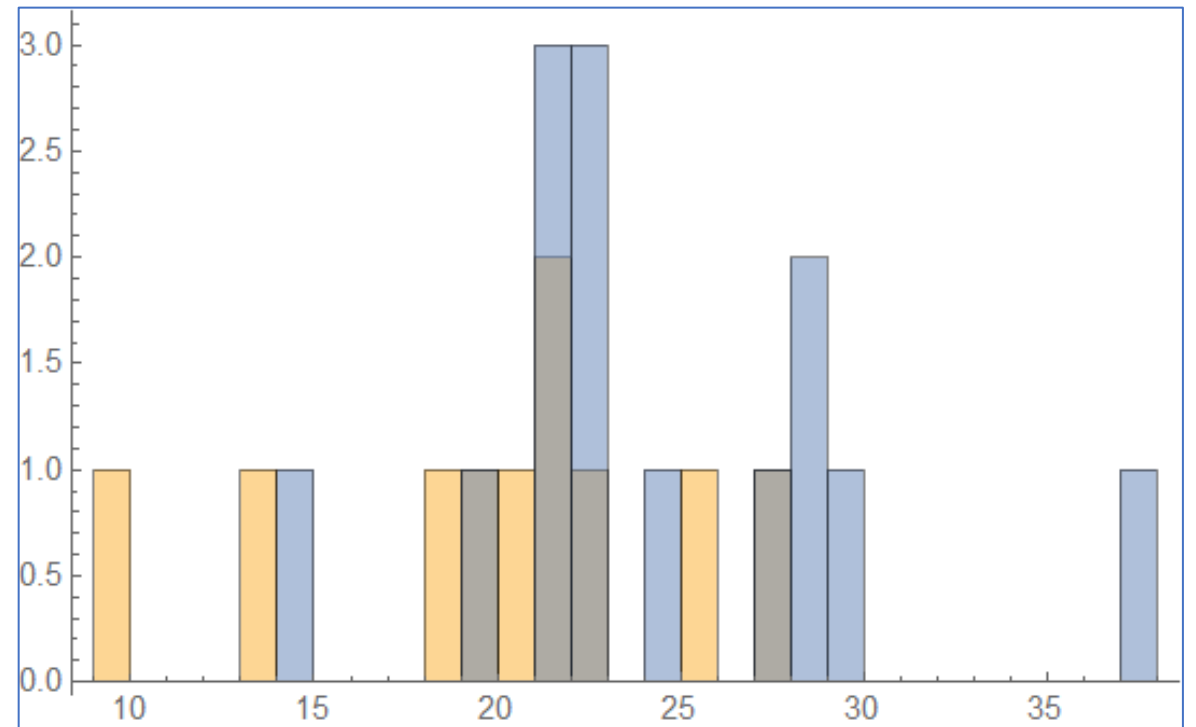
### The non parametric test

is less powerful than the parametric test when the shape of the distribution is known exactly, but useful when the shape of the distribution is not known.

# 例7.5 2 sample problem

引用：東京大学教養学部統計学教室編：「自然科学の統計学」、東京大学出版会、1992年。

- The two population distributions are the same ?
- $\text{data1} = \{9.5, 13.9, 18.1, 19.9, 20.6, 21.5, 21.8, 22.1, 25.7, 27.9\}$ ;  $m=10$
- $\text{data2} = \{14.7, 19.8, 21.3, 21.6, 21.7, 22.2, 22.4, 22.7, 24.6, 27.1, 28.0, 28.0, 29.3, 37.1\}$ ;  $n=14$



# Inferential statistics

Two inferred population distributions are independent ???

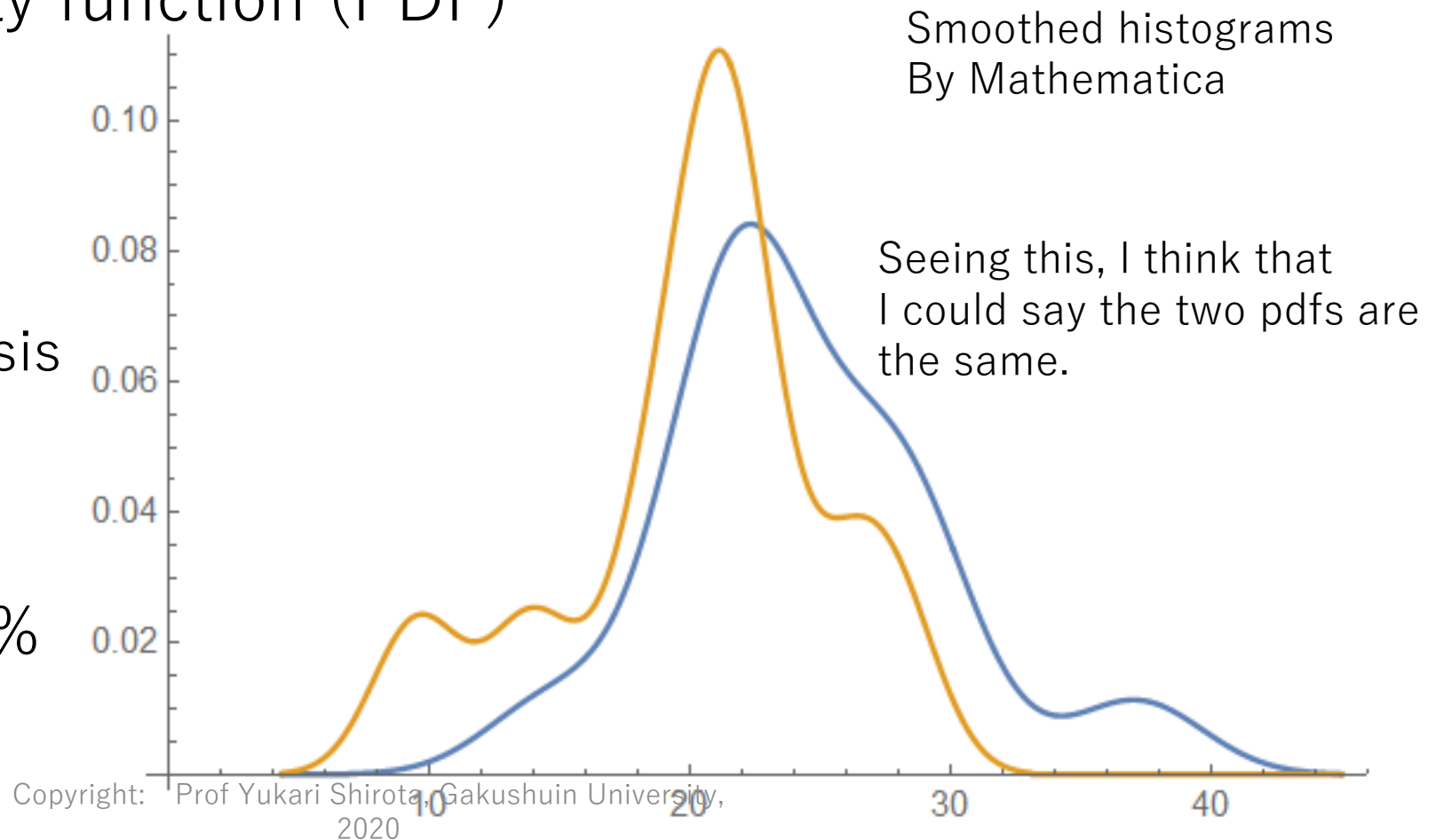
- Probability density function (PDF)

- $f(x) = g(x) ?$

- Null Hypothesis  
 $f(x) = g(x)$

- Alternative Hypothesis  
 $f(x) \neq g(x)$

Significant level: 5%



Ranking 1,2,3,⋯, 24

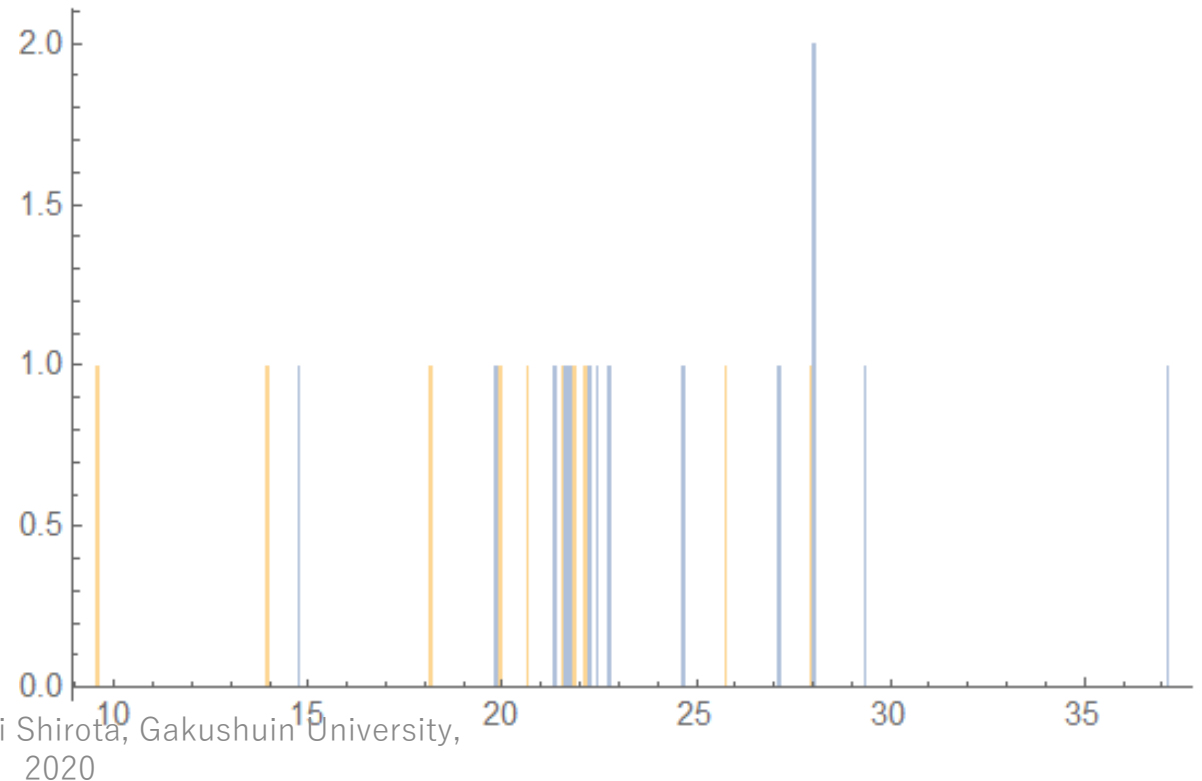
## Rank-sum test (Wilcoxon test, Mann-Whitney test)

- $\{9.5, 1\}, \{13.9, 1\}, \{14.7, 2\}, \{18.1, 1\}, \{19.8, 2\}, \{19.9, 1\}, \{20.6, 1\}, \{21.3, 2\}, \{21.5, 1\}, \{21.6, 2\}, \{21.7, 2\}, \{21.8, 1\}, \{22.1, 1\}, \{22.2, 2\}, \{22.4, 2\}, \{22.7, 2\}, \{24.6, 2\}, \{25.7, 1\}, \{27.1, 2\}, \{27.9, 1\}, \{28., 2\}, \{28., 2\}, \{29.3, 2\}, \{37.1, 2\}$

- Sum of group1 member's ranks

$$\begin{aligned} W &= \text{Sum}[\{1, 2, 4, 6, 7, 9, 12, 13, 18, 20\}] \\ &= 92 \end{aligned}$$

If the group1's peak shifts to the right,  
Rank sum  $W$  also increases



# Rank-sum test (Wilcoxon test, Mann-Whitney test)

According to statistical theory,  
the distribution of the rank sum  $W$  is known.  
We use it.

Rank sum  $W$  approximately follows the distribution

$$N\left(\frac{m(m+n+1)}{2}, \frac{mn(m+n+1)}{12}\right)$$

Where  $m$  is the group 1's size and  $n$  is the group 2's size

課題：この計算を自分で行う。結論も書く。

$$N\left(\frac{m(m+n+1)}{2}, \frac{mn(m+n+1)}{12}\right)$$

- $m = 10$
- $n = 14$
- $\frac{m(m+n+1)}{2} = 125$

$$\frac{mn(m+n+1)}{12} = 291.7$$

- $W = 92$  The z value ?

$$z = \frac{92 - 125}{\sqrt{291.7}} = -1.9$$

The z value falls in the non-rejection region.

Make the decision:

The two distributions are the same.

