ラグランジュ未定乗数法ドリル問題集

作成　白田由香利

学籍番号________________________　　　氏名__________________________

(Q1) CAT-DOG company produces and sells two kinds of goods, A and B. Let x and y be the respective production quantities. Find the total production cost C when the production cost is minimized under the constraint that the sum of both production quantities is 72.

C=12x^2-6xy+6y^2

$$C(x, y) = 12x^2 - 6xy + 6y^2$$

In the problem, approximate how much the production cost increases when the number of constraints is increased to 73 from the optimal solution C.

ANSWER:

Given Data:

1. **Production quantities**: x (quantity of good A) and y (quantity of good B).
2. **Total production cost function**:

$$C = 12x^2 - 6xy + 6y^2$$

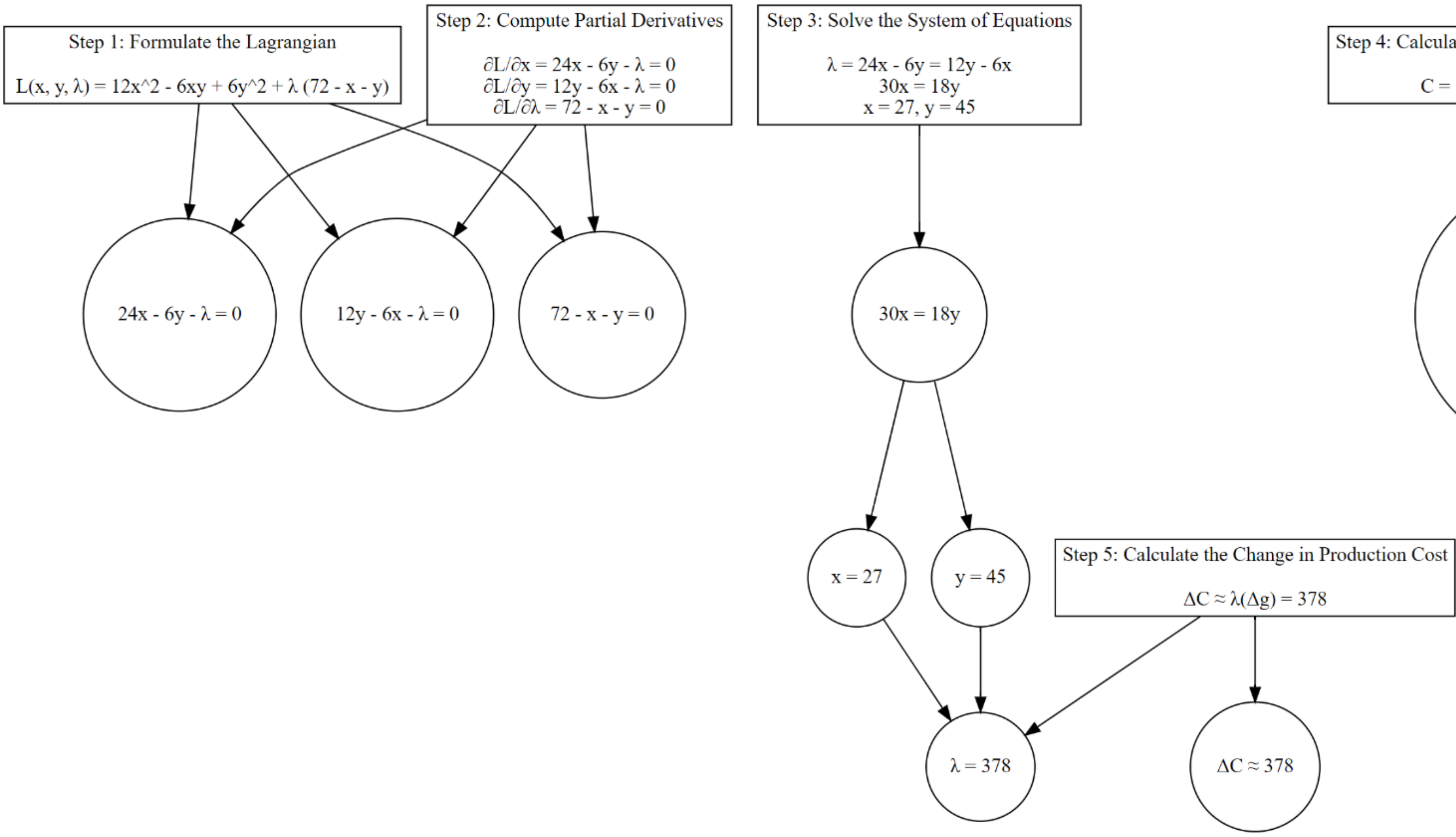
3. **Constraint**: The sum of both production quantities is 72, i.e.,

$$x + y = 72$$

Unknown:

- The minimized total production cost C under the given constraint.

So, the problem is to find the value of C when the production quantities x and y are optimized under the constraint $x + y = 72$.



The change in the constraint Δg is:

$\Delta g = 73 - 72 = 1$

The approximate change in the objective function (production cost) ΔC can be estimated using λ:

$\Delta C \approx \lambda \cdot \Delta g = 378 \cdot 1 = 378$

So, the approximate increase in production cost is:

$\Delta C \approx 378$

Therefore, when the constraint changes from $x + y = 72$ to $x + y = 73$, the production cost is expected to increase by approximately 378.

(Q2)

Given the production function as follows, find the point Q at which Q is maximized under the constraint that 2K + L = 200. From the point of the optimal solution, approximate how much Q will increase if 200 in the equation is increased to 201.

$$Q(K,L) = K^{0.6}L^{0.3}$$

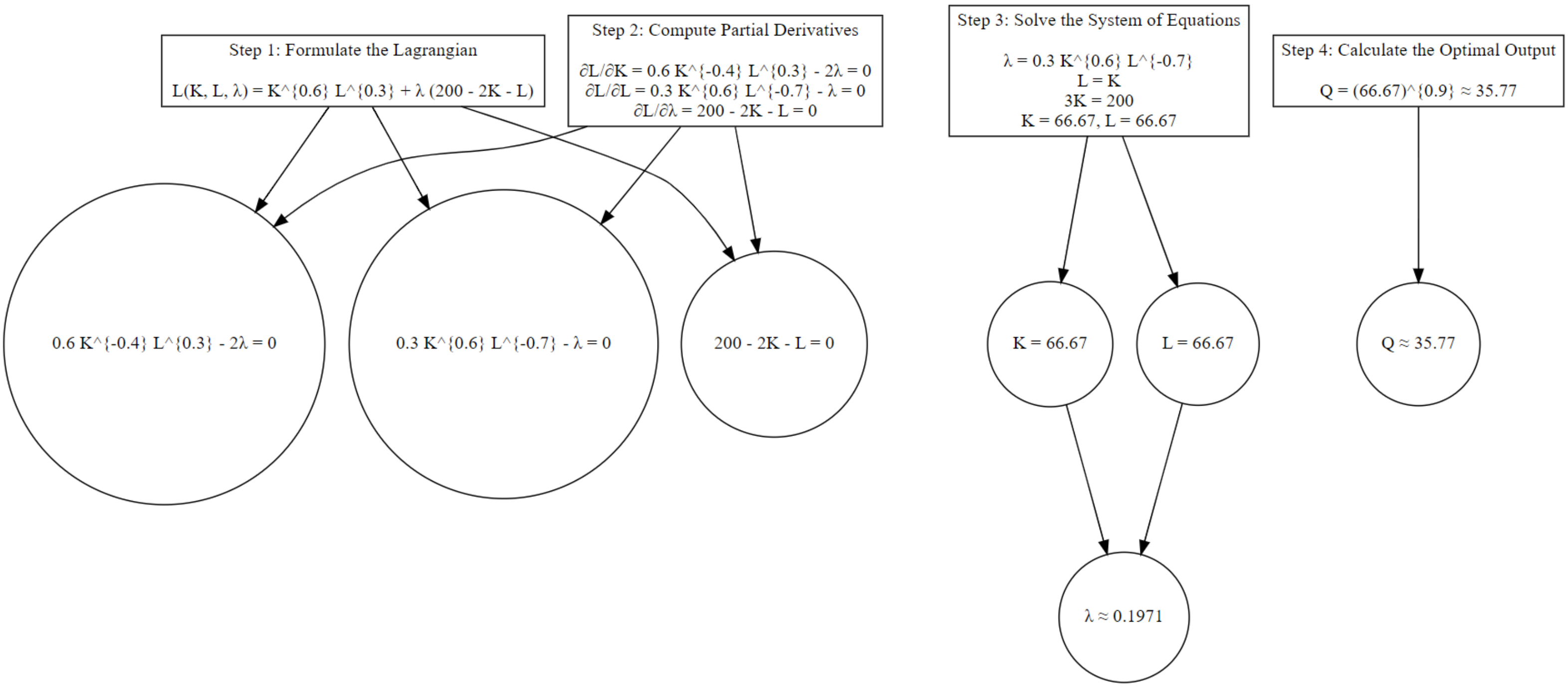
ANSWER:

Given Data

1. **Production function**: $Q = K^{0.6}L^{0.3}$
2. **Constraint**: $2K + L = 200$

Unknowns

1. **Optimal production quantities**:
   - K: Capital input
   - L: Labor input
2. **Maximum production output** Q at the optimal point.
3. **Change in production output** ΔQ when the constraint changes from 200 to 201.



(Q3) The prices of two kinds of goods A and B are 100 yen and 200 yen. The budgeted amount is 3000 yen. Let x and y be the respective production quantities. We want to maximize utility by purchasing A and B using up the budgeted amount. The utility function u is given by

$$u(x, y) = \sqrt{x} + \sqrt{y}$$

Find the number of goods A that maximizes utility. Also, approximate how much the utility would increase if the budget were increased by one more yen at that point.

ANSWER:

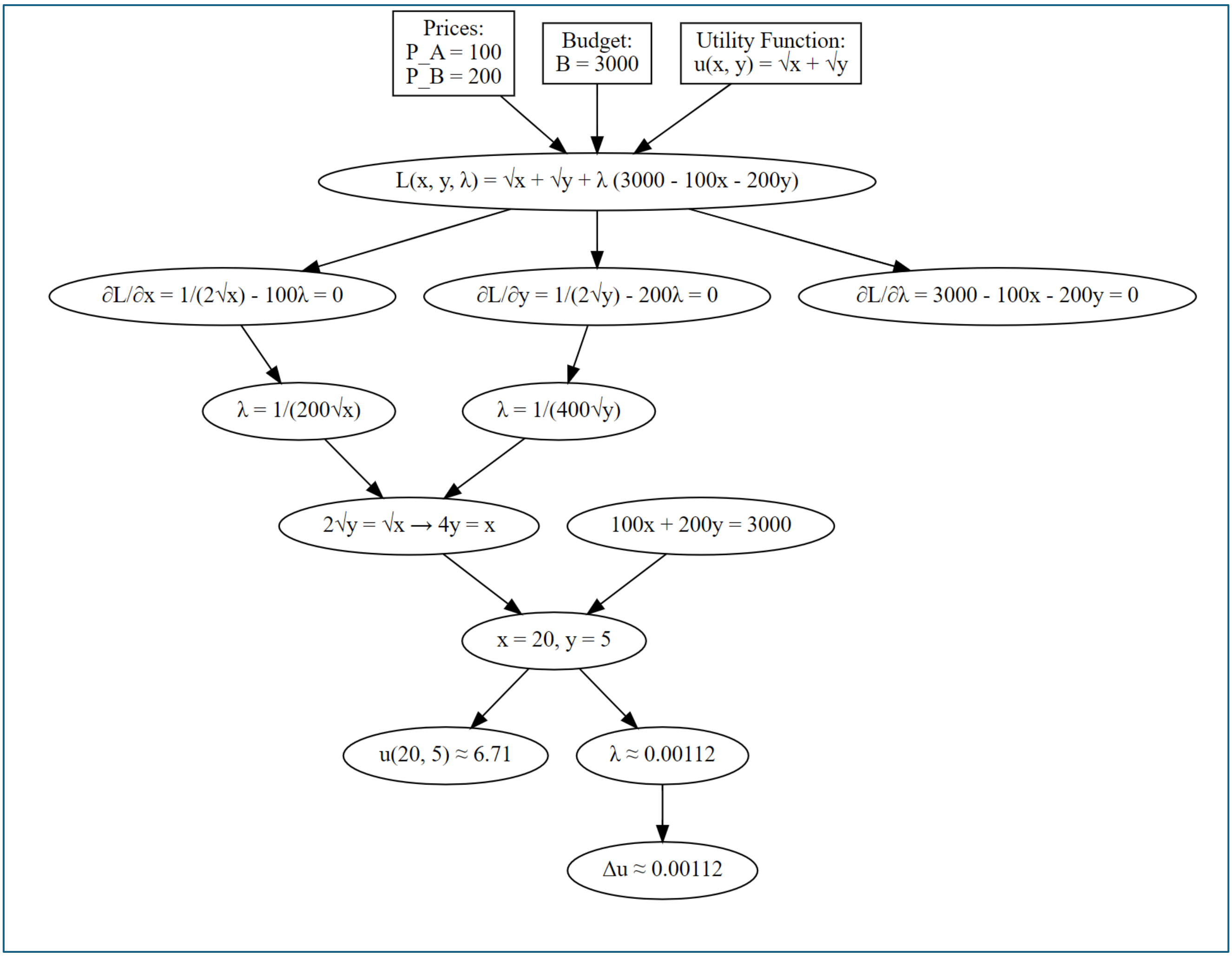
Given Data

1. **Prices of goods**:
    - Price of A: $P_A = 100$ yen
    - Price of B: $P_B = 200$ yen
2. **Budget**: $B = 3000$ yen
3. **Utility function**:

$$u(x, y) = \sqrt{x} + \sqrt{y}$$

Unknowns

1. **Optimal production quantities**:
    - x: Quantity of good A
    - y: Quantity of good B
2. **Maximum utility** u
3. **Change in utility** Δu when the budget increases by one yen



(Q4)

Under the condition x^2+y^2=1, find x and y values which maximize the following z(x, y)

z(x, y) = 9x^2 + 9y^2 +8x*y.

ANSWER:

Given Data

1. **Function to maximize:**

$$z(x, y) = 9x^2 + 9y^2 + 8xy$$

2. **Constraint:**

$$x^2 + y^2 = 1$$

Unknowns

1. **Optimal values of x and y** that maximize $z(x, y)$