Gibbs Update
Modification Operation in MCMC

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http://www-cc.gakushuin.ac.jp/~20010570/mathABC/SELECTED/

Suppose that x is a sample from the distribution P.
Conduct Gibbs update on x and we get the new x’ .
As x’ also follows P, then X’ is also a sample from P.

$$\sum_{x_i}\{P(x_i'|x_1,\cdots,x_{i-1}, \quad x_{i+1},\cdots,x_N)\times$$

$$\underline{P(x_1,\cdots,x_{i-1},x_i,x_{i+1},\cdots,x_N)}\}$$

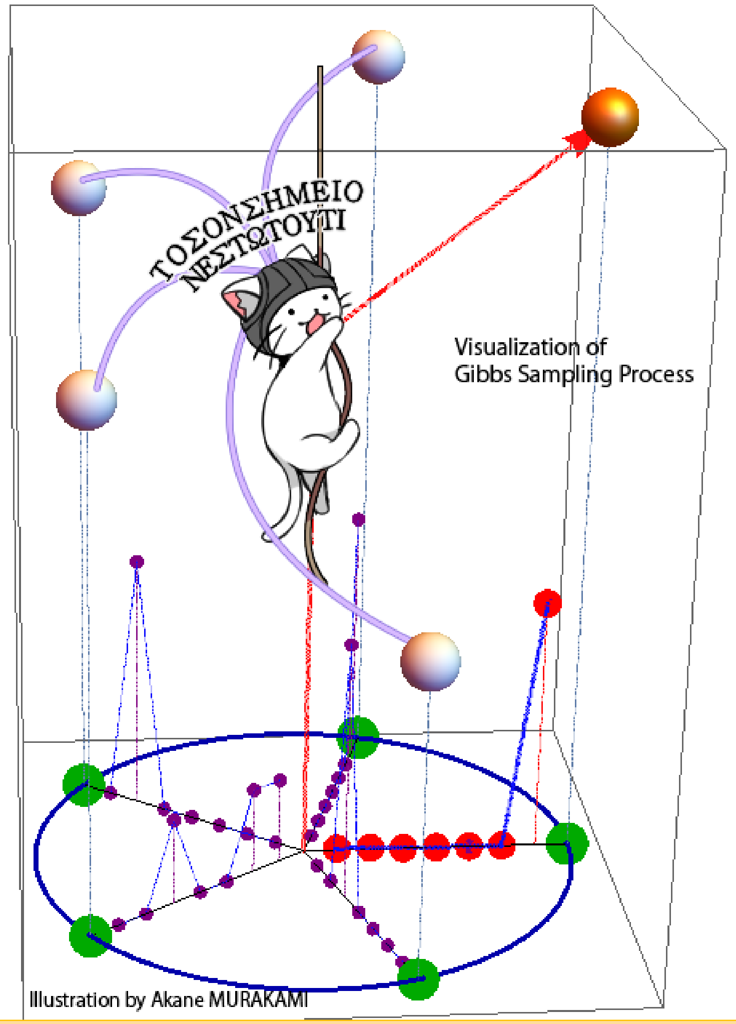
$$= P(x_i'|x_1,\cdots,x_{i-1}, \quad x_{i+1},\cdots,x_N)$$

$$\times P(x_1,\cdots,x_{i-1}, \quad x_{i+1},\cdots,x_N)$$

$$= \underline{P(x_1,\cdots,x_{i-1},x_i',x_{i+1},\cdots,x_N)}$$

Joint probability
$p(A,B)$
$= p(A|B)p(B)$

$$P(z_d = k\,|W, z_{\backslash d},\ \alpha,\ \beta)$$



We would like to make a Markov chain of which stationary distribution is the posterior.

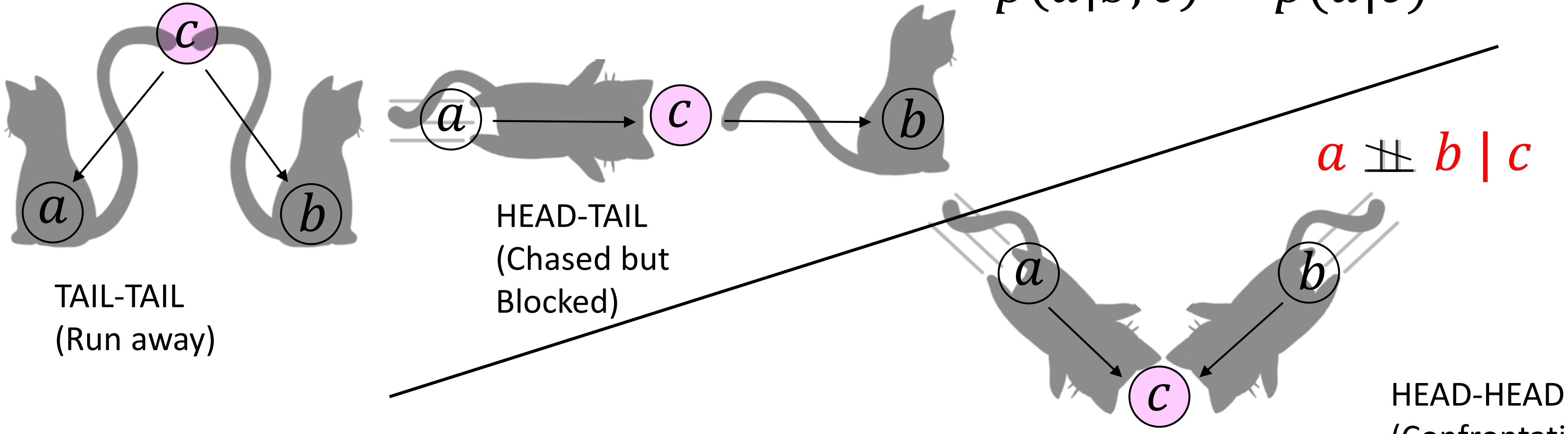
Conditional Independence

Given c, a and b are independent under the condition c.

$a \perp\!\!\!\perp b \mid c$

$$p(a, b|c) = p(a|c)\,p(b|c)$$

$$p(a|b, c) = p(a|c)$$



TAIL-TAIL
(Run away)

HEAD-TAIL
(Chased but
Blocked)

HEAD-HEAD
(Confrontation)

Illustrations by Akane MURAKAMI

Deductive Reasoning Diagram for a posterior for d-th document belongs to topic k

$$p\left(z_d = k \mid \boldsymbol{W}, \boldsymbol{z}_{\setminus d}, \alpha, \beta\right)$$

Bayes

$$\frac{p\left(z_d = k, \boldsymbol{W}, \boldsymbol{z}_{\setminus d}, \alpha, \beta\right)}{p\left(\boldsymbol{W}, \boldsymbol{z}_{\setminus d}, \alpha, \beta\right)}$$

Joint $p(A, B) = p(A|B)p(B)$

$$\frac{p\left(z_d = k, \boldsymbol{W}, \boldsymbol{z}_{\setminus d} \mid \alpha, \beta\right) p(\alpha, \beta)}{p\left(\boldsymbol{W}, \boldsymbol{z}_{\setminus d} \mid \alpha, \beta\right) p(\alpha, \beta)}$$

$$\frac{p\left(z_d = k, \boldsymbol{W}, \boldsymbol{z}_{\setminus d} \mid \alpha, \beta\right)}{p\left(\boldsymbol{W}, \boldsymbol{z}_{\setminus d} \mid \alpha, \beta\right)}$$

No z_d exists

$$\propto \; p\left(z_d = k, \boldsymbol{W}, \boldsymbol{z}_{\setminus d} \mid \alpha, \beta\right)$$

Bayes

$$\frac{p\left(z_d = k, \boldsymbol{W}, \boldsymbol{z}_{\setminus d}, \alpha, \beta\right)}{p(\alpha, \beta)}$$

Joint

Joint

$$\frac{p(\alpha, \beta)\, p\left(z_d = k, \boldsymbol{W}_{\setminus d}, \boldsymbol{z}_{\setminus d} \mid \alpha, \beta\right)}{p(\alpha, \beta)} \times p\left(\boldsymbol{w}_d \mid z_d = k, \boldsymbol{W}_{\setminus d}, \boldsymbol{z}_{\setminus d}, \alpha, \beta\right)$$

$$p\left(z_d = k, \boldsymbol{W}_{\setminus d}, \boldsymbol{z}_{\setminus d} \mid \alpha, \beta\right) \quad p\left(\boldsymbol{w}_d \mid z_d = k, \boldsymbol{W}_{\setminus d}, \boldsymbol{z}_{\setminus d}, \alpha, \beta\right)$$

Joint

$$p\left(z_d = k \mid \boldsymbol{W}_{\setminus d}, \boldsymbol{z}_{\setminus d}, \alpha, \beta\right) p\left(\boldsymbol{W}_{\setminus d}, \boldsymbol{z}_{\setminus d} \mid \alpha, \beta\right)$$

No z_d exists

$$\propto \; p\left(z_d = k \mid \boldsymbol{W}_{\setminus d}, \boldsymbol{z}_{\setminus d}, \alpha, \beta\right)$$

Bayes

$$\frac{p\left(\boldsymbol{w}_d, \boldsymbol{W}_{\setminus d}, z_d = k, \boldsymbol{z}_{\setminus d}, \alpha, \beta\right)}{p\left(z_d = k, \boldsymbol{W}_{\setminus d}, \boldsymbol{z}_{\setminus d}, \alpha, \beta\right)}$$

$$\frac{\dfrac{p\left(\boldsymbol{w}_d, \boldsymbol{W}_{\setminus d}, z_d = k, \boldsymbol{z}_{\setminus d}, \alpha, \beta\right)}{p\left(z_d = k, \boldsymbol{z}_{\setminus d}, \alpha, \beta\right)}}{\dfrac{p\left(\boldsymbol{W}_{\setminus d}, z_d = k, \boldsymbol{z}_{\setminus d}, \alpha, \beta\right)}{p\left(z_d = k, \boldsymbol{z}_{\setminus d}, \alpha, \beta\right)}}$$

Bayes

$$\frac{p\left(\boldsymbol{w}_d, \boldsymbol{W}_{\setminus d} \mid z_d = k, \boldsymbol{z}_{\setminus d}, \alpha, \beta\right)}{p\left(\boldsymbol{W}_{\setminus d} \mid z_d = k, \boldsymbol{z}_{\setminus d}, \alpha, \beta\right)}$$

$$\frac{p(\boldsymbol{w}_d, \boldsymbol{W}_{\backslash d} | z_d = k, \boldsymbol{z}_{\backslash d}, \alpha, \beta)}{p(\boldsymbol{W}_{\backslash d} | z_d = k, \boldsymbol{z}_{\backslash d}, \alpha, \beta)}$$

$\boldsymbol{W} = \boldsymbol{W}_{\backslash d} \cup \{\boldsymbol{w}_d\}$

$$\frac{p(\boldsymbol{W} \ | z_d = k, \boldsymbol{z}_{\backslash d}, \alpha, \beta)}{p(\boldsymbol{W}_{\backslash d} | z_d = k, \boldsymbol{z}_{\backslash d}, \alpha, \beta)}$$

$\boldsymbol{W}_{\backslash d} \perp\!\!\!\perp \alpha \mid \boldsymbol{z}_{\backslash d}, \beta$

$\boldsymbol{W}_{\backslash d} \perp\!\!\!\perp z_d \mid \boldsymbol{z}_{\backslash d}, \beta$

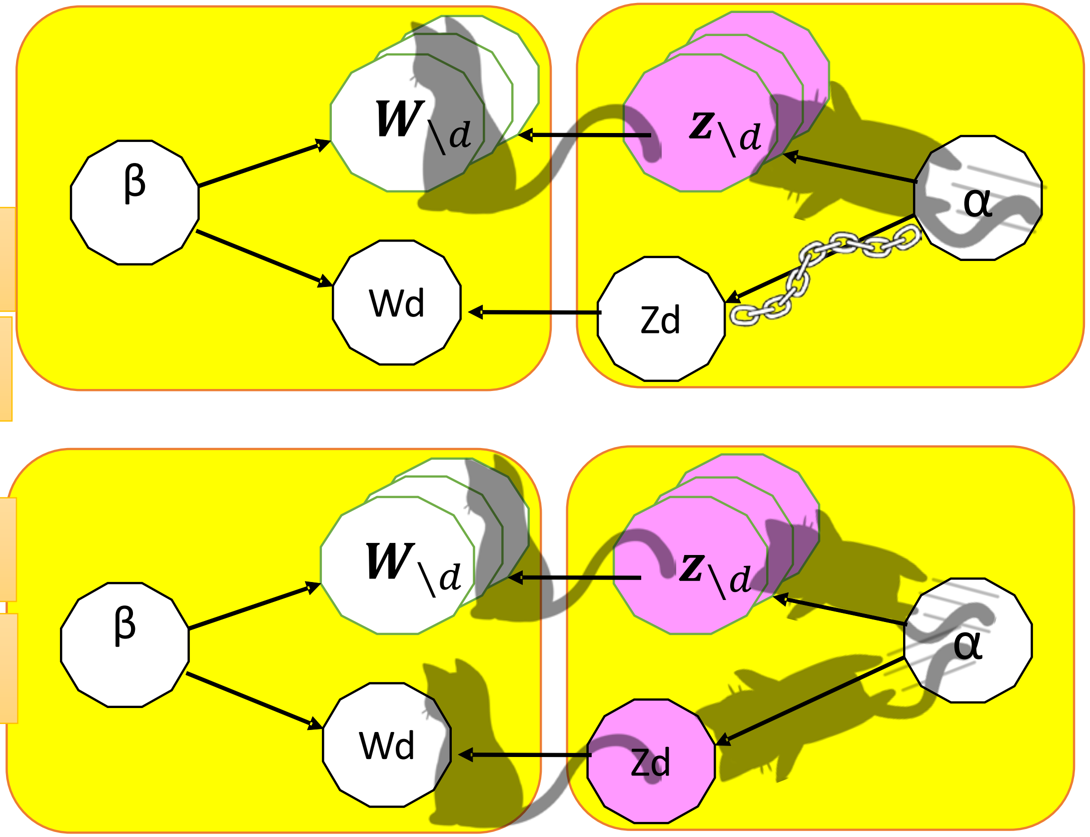
$$\frac{p(\boldsymbol{W} \ | z_d = k, \boldsymbol{z}_{\backslash d}, \alpha, \beta)}{p(\boldsymbol{W}_{\backslash d} | \ \boldsymbol{z}_{\backslash d}, \beta)}$$

$\boldsymbol{W}_{\backslash d} \perp\!\!\!\perp \alpha \mid \boldsymbol{z}_{\backslash d}, \beta$

$\boldsymbol{W}_d \perp\!\!\!\perp \alpha \mid \boldsymbol{z}_d, \beta$

$\boldsymbol{W} = \boldsymbol{W}_{\backslash d} \cup \{\boldsymbol{w}_d\}$

$$\frac{p(\boldsymbol{W} \ | z_d = k, \boldsymbol{z}_{\backslash d}, \beta)}{p(\boldsymbol{W}_{\backslash d} | \ \boldsymbol{z}_{\backslash d}, \beta)}$$



Illustrations by Akane MURAKAMI

Posterior for d-th document belongs to topic k

$$p(z_d = k | \boldsymbol{W}, \boldsymbol{z}_{\backslash d}, \alpha, \beta) \propto p(z_d = k | \boldsymbol{W}_{\backslash d}, \boldsymbol{z}_{\backslash d}, \alpha, \beta) \frac{p(\boldsymbol{W} \ | z_d = k, \boldsymbol{z}_{\backslash d}, \beta)}{p(\boldsymbol{W}_{\backslash d} | \ \boldsymbol{z}_{\backslash d}, \beta)}$$