

# 構造化学

晶系とBravais格子

第9回 6月11日

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# 前回の復習

対称要素と点群について述べました。

○対称要素

$p$ 回回転軸、 $p$ 回回反軸、対称心、鏡面

○特定の対称要素の組み合わせが群をなす

○結晶に表れる点群は32種類

# 本日の目標

**Bravais格子**について理解しよう

内容

- 平面格子
- 空間格子
- 7つの晶系
- Bravais格子

# 対称性

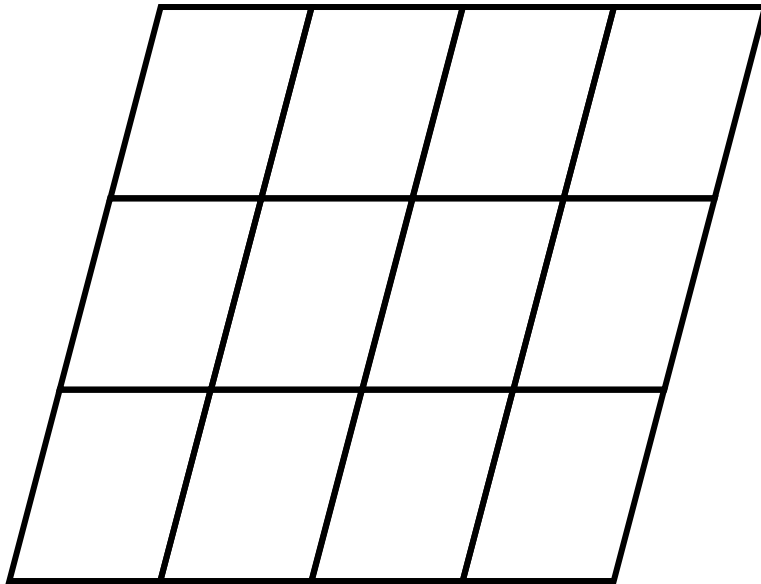
点の回りの対称(1次元): 32種類の点群

平面格子の単位胞の対称(2次元): 5種類

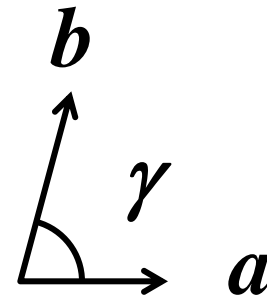
空間格子の単位胞の対称(3次元): 7種類(7晶系)

7晶系 + 複合格子 = 格子

# 平面格子

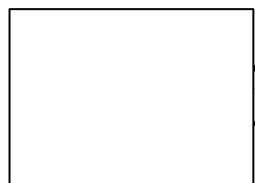


パラメータ



2回軸以上の対称性をもつ  
平面格子を考える

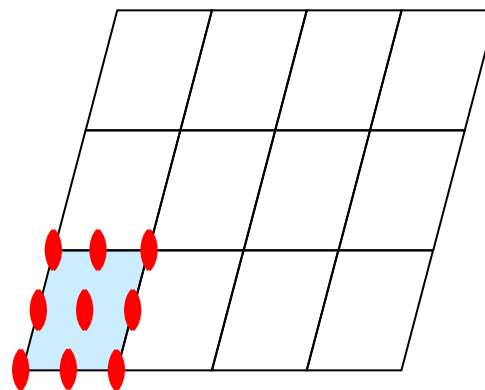
# 平面格子(1)



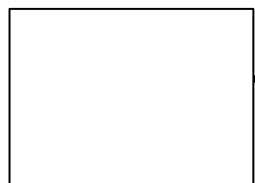
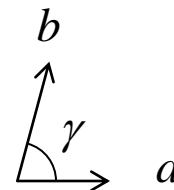
格子(oblique net)

$$a \neq b$$

$\gamma$  任意



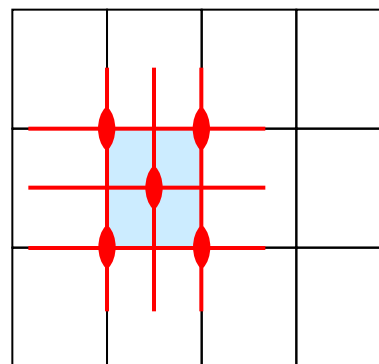
2



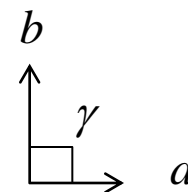
格子(rectangular net)

$$a \neq b$$

$$\gamma = 90^\circ$$



$m$



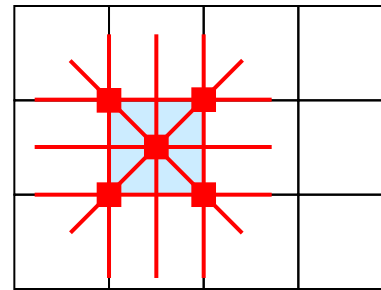
# 平面格子(2)



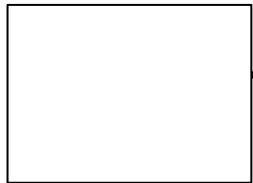
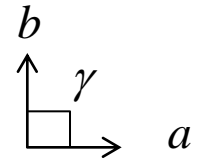
格子(square net)

$$a = b$$

$$\gamma = 90^\circ$$



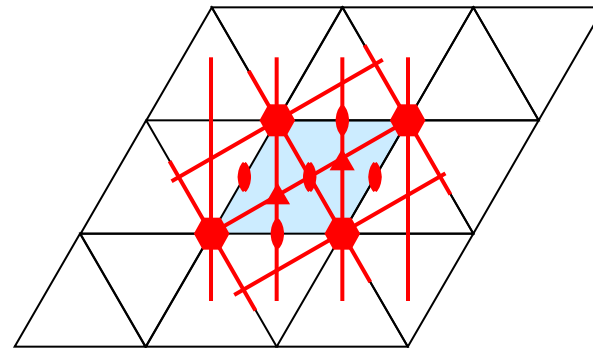
4



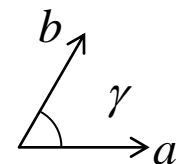
格子(hexagonal net)

$$a = b$$

$$\gamma = 60^\circ, 120^\circ$$



6



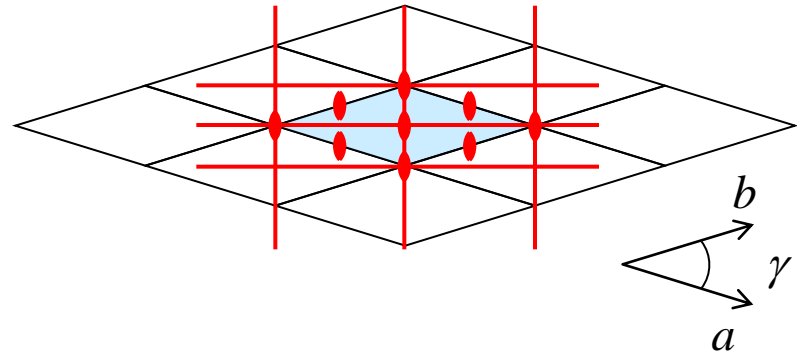
# 平面格子(3)



格子(rhombic net)

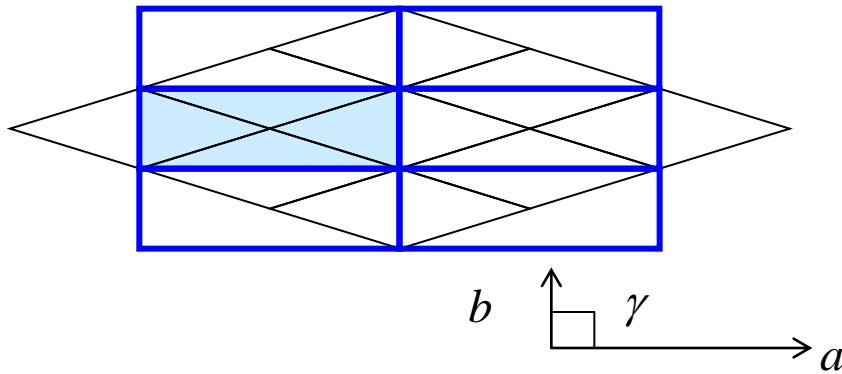
$$a = b$$

$\gamma$  任意



2

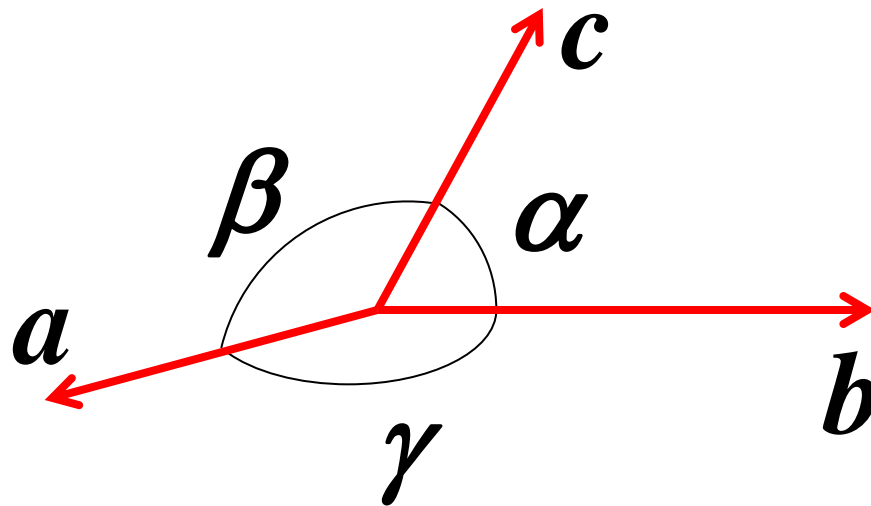
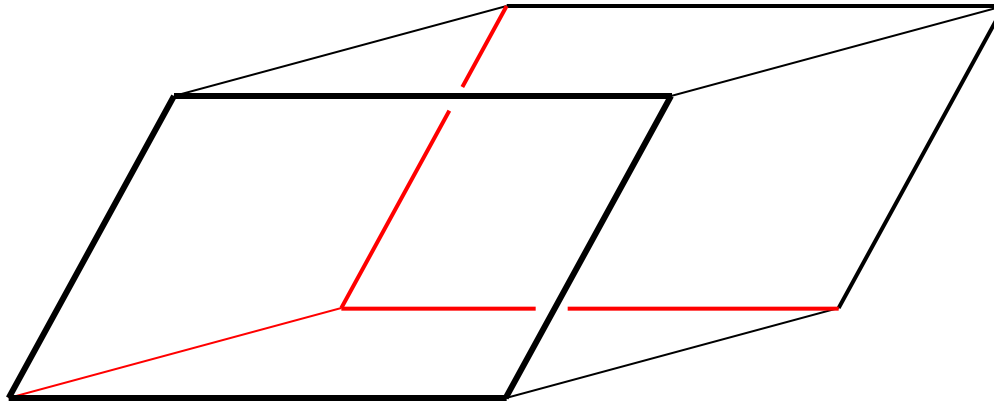
面心長方格子ともいえる



基本単位格子ではないが、  
対称性をよく表している。  
→Bravais格子の考え方



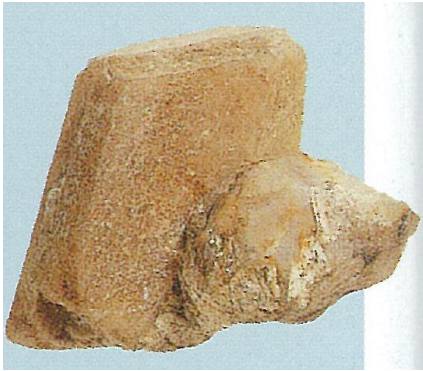
# 空間格子のパラメータ



# 7晶系

晶系	必須の対称要素	単位胞の形状
三斜	なし	$a \neq b \neq c, \alpha \neq \beta \neq \gamma$ ( $c < a < b$ )
単斜	1本の2回軸	$a \neq b \neq c, \alpha = \gamma = 90^\circ, \beta \neq 90^\circ$ ( $c < a$ )
斜方	3本の2回軸	$a \neq b \neq c, \alpha = \beta = \gamma = 90^\circ$ ( $c < a < b$ )
正方	1本の4回軸	$a = b \neq c, \alpha = \beta = \gamma = 90^\circ$
三方 六方 菱面	1本の3回軸	$a = b \neq c, \alpha = \beta = 90^\circ, \gamma = 120^\circ$ $a = b = c, \alpha = \beta = \gamma \neq 90^\circ$
六方	1本の6回軸	$a = b \neq c, \alpha = \beta = 90^\circ, \gamma = 120^\circ$
立方	4本の3回軸	$a = b = c, \alpha = \beta = \gamma = 90^\circ$

# 7つの晶系



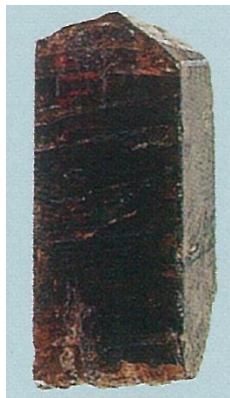
三斜



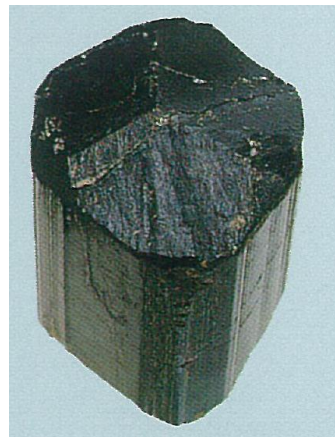
単斜



斜方



正方



三方



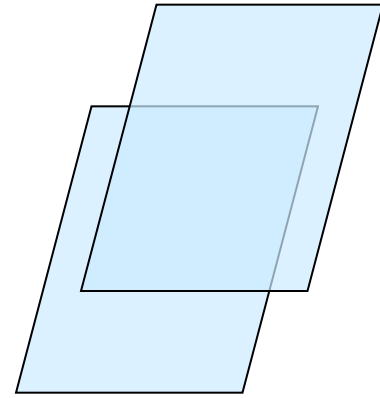
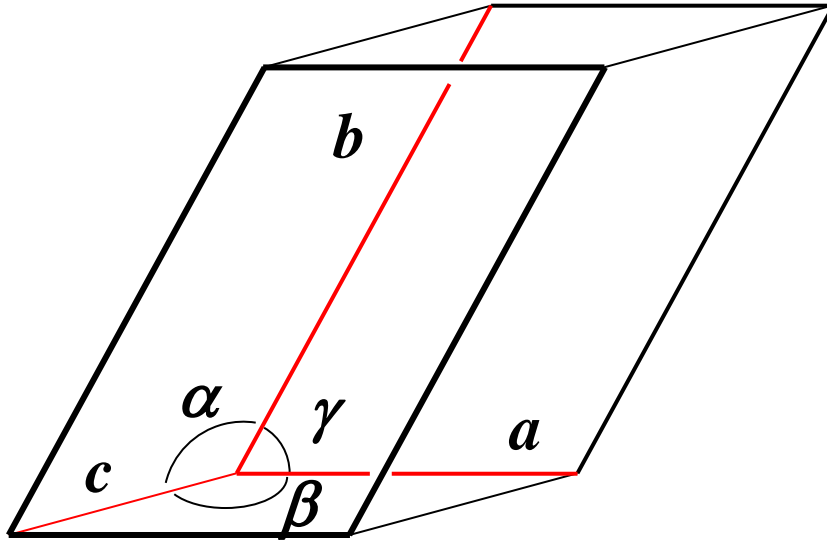
六方



立方



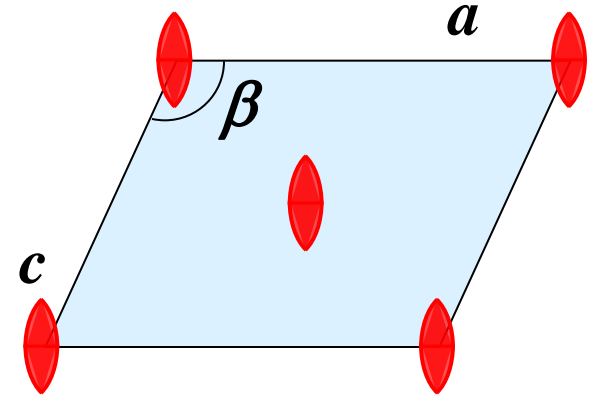
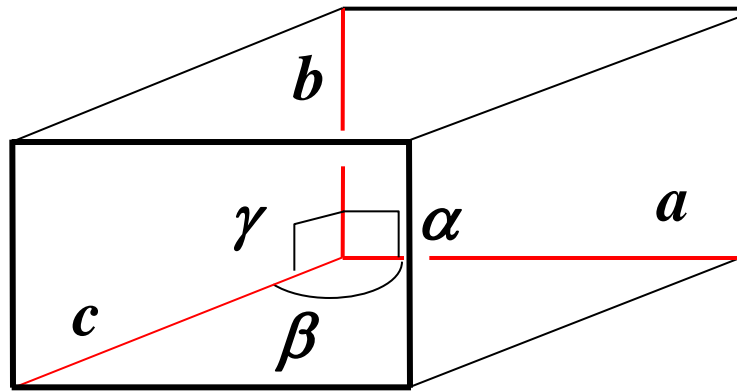
# 晶系 Triclinic



$$a \neq b \neq c, \quad \alpha \neq \beta \neq \gamma$$
$$(c < a < b)$$

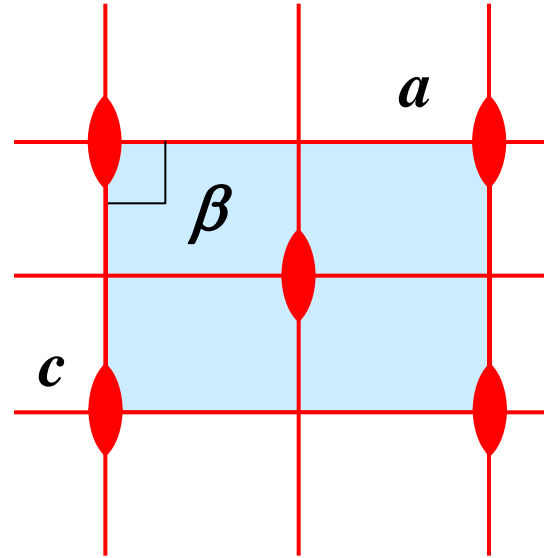
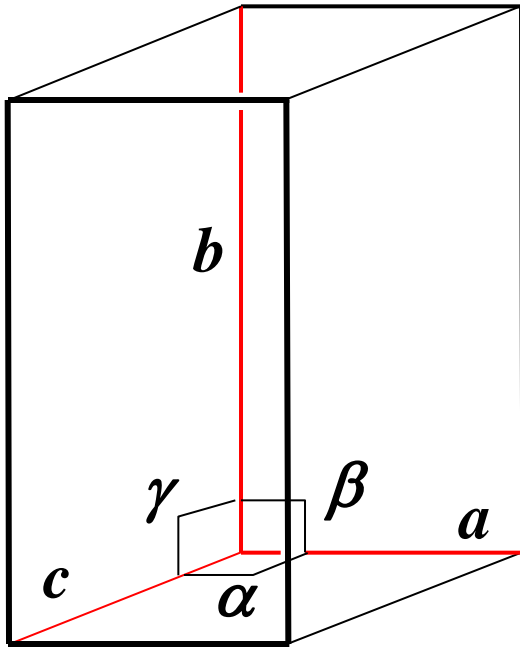


# 晶系 Monoclinic



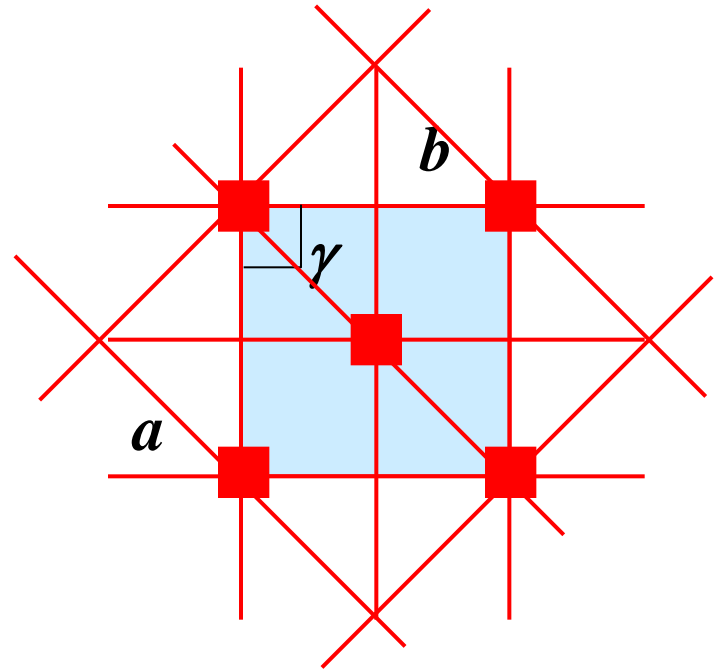
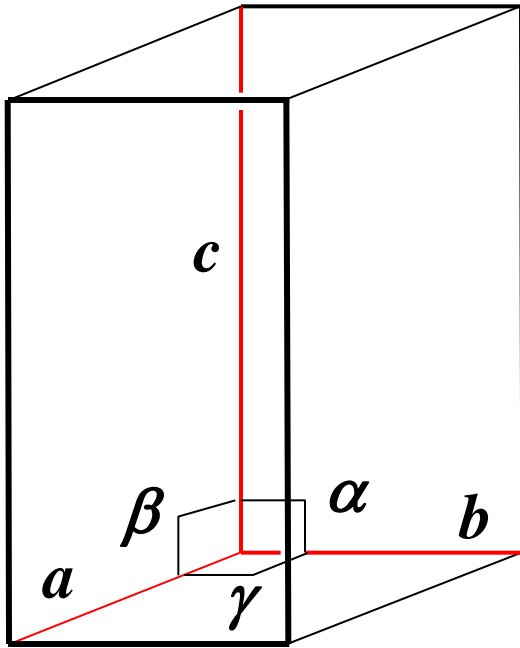
$$a \neq b \neq c, \quad \alpha = \gamma = 90^\circ, \quad \beta \neq 90^\circ$$
$$(c < a)$$

# 晶系 Orthorhombic



$$a \neq b \neq c, \quad \alpha = \beta = \gamma = 90^\circ$$
$$(c < a < b)$$

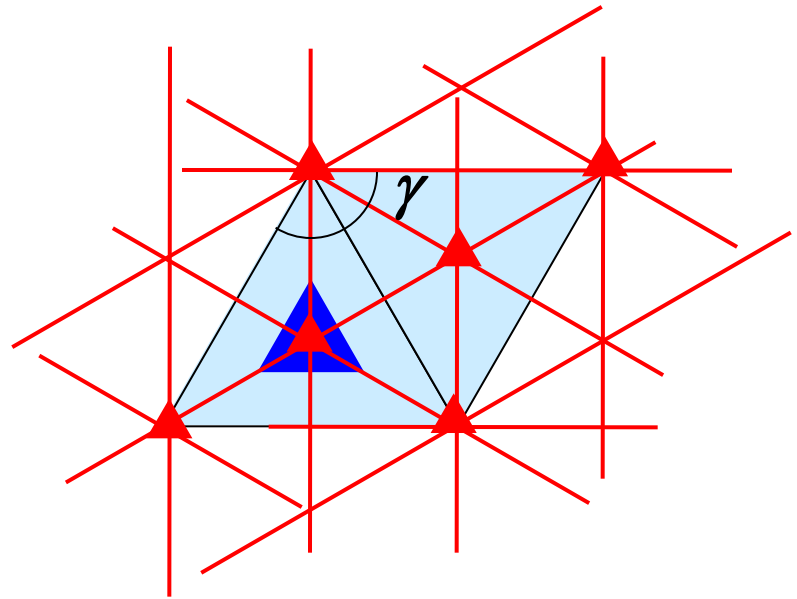
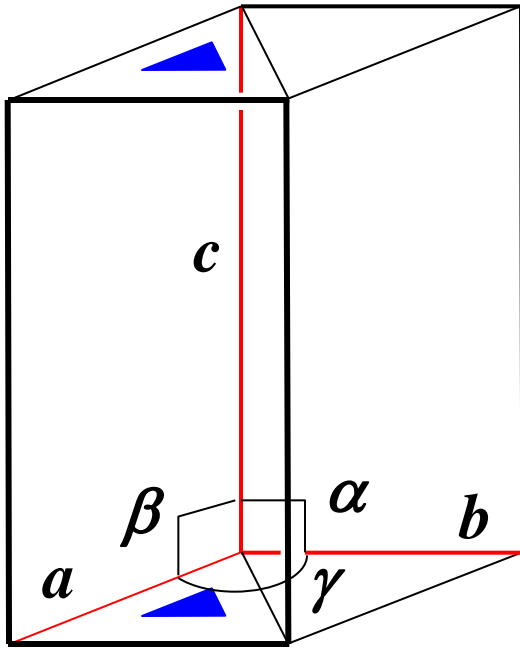
# 晶系 Tetragonal



$$a = b \neq c, \quad \alpha = \beta = \gamma = 90^\circ$$

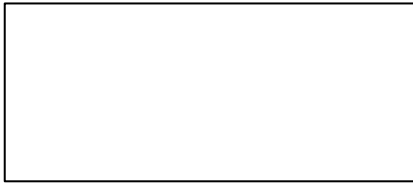


# 晶系 Trigonal

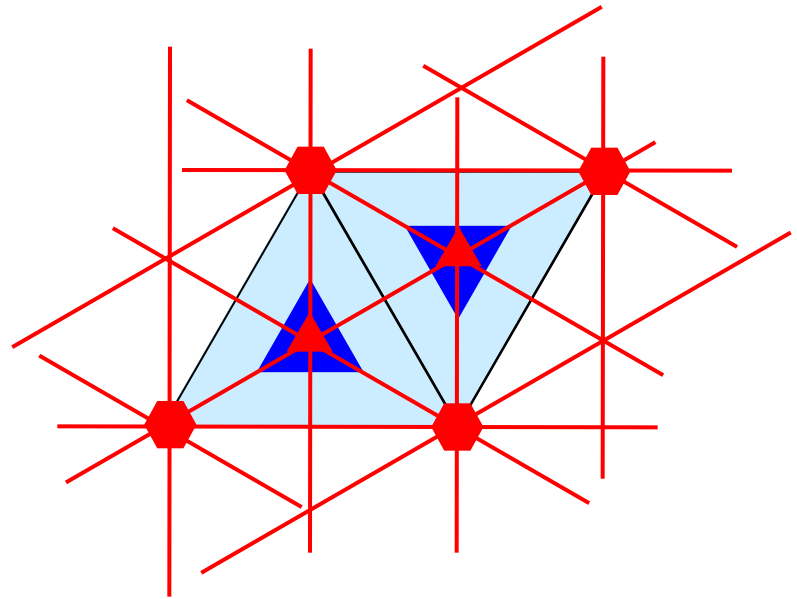
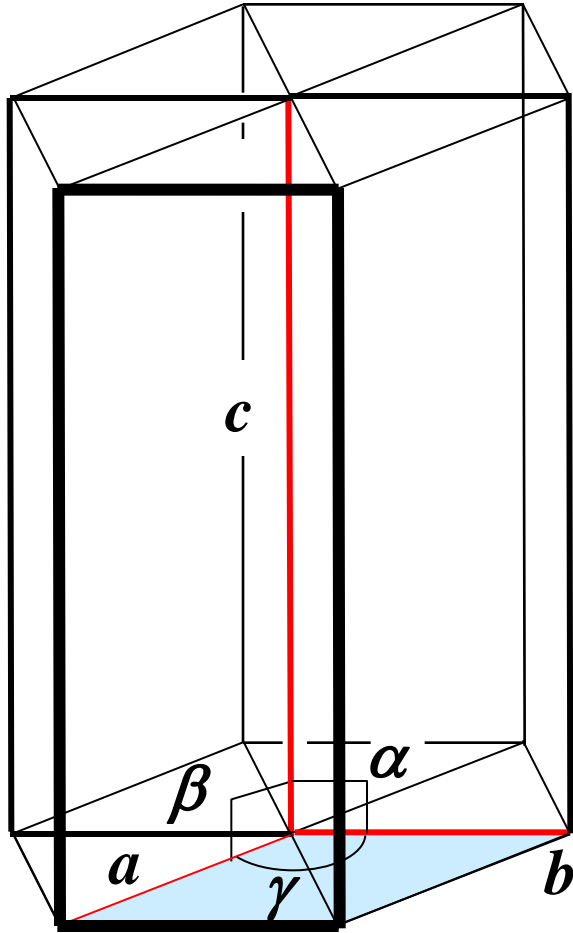


$$a = b \neq c, \quad \alpha = \beta = 90^\circ, \quad \gamma = 120^\circ$$





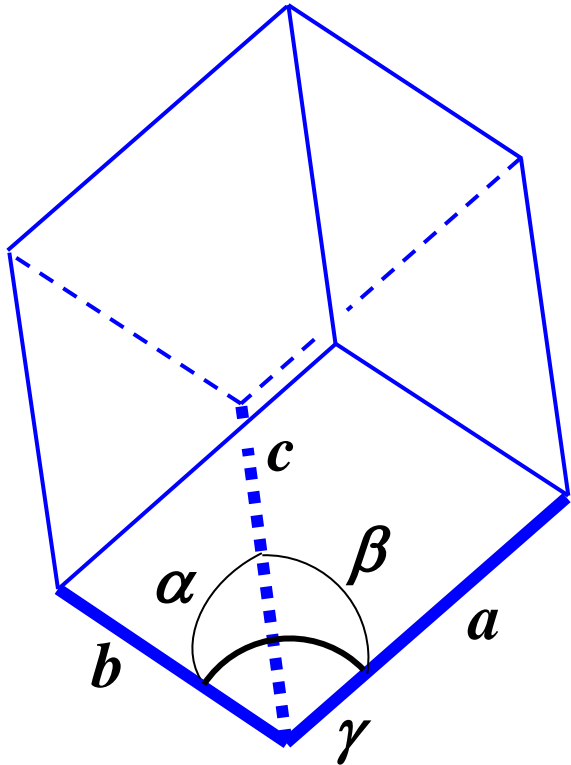
# 晶系 Hexagonal



$$a = b \neq c,$$

$$\alpha = \beta = 90^\circ, \gamma = 120^\circ$$

# 晶系 Rhombohedral

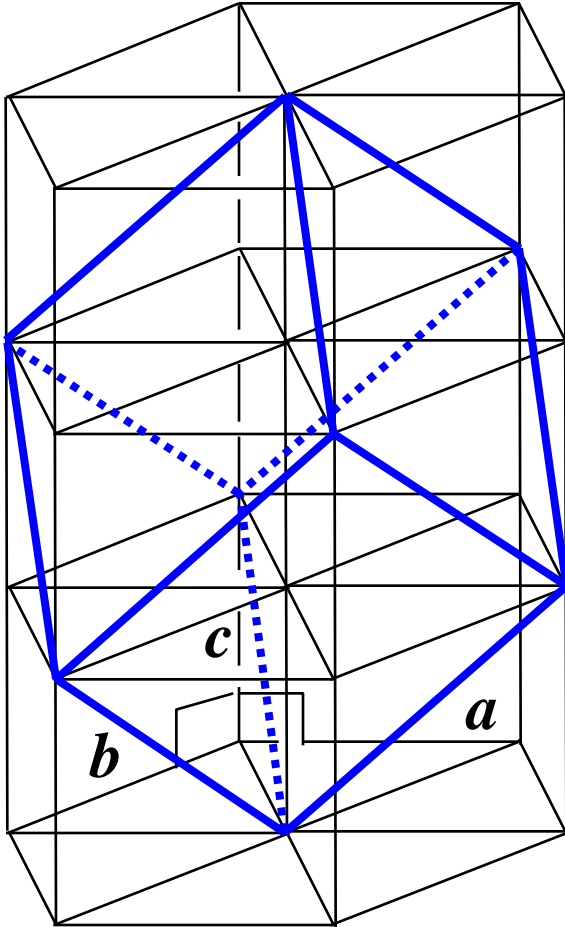


$$a = b = c,$$

$$\alpha = \beta = \gamma \neq 90^\circ$$

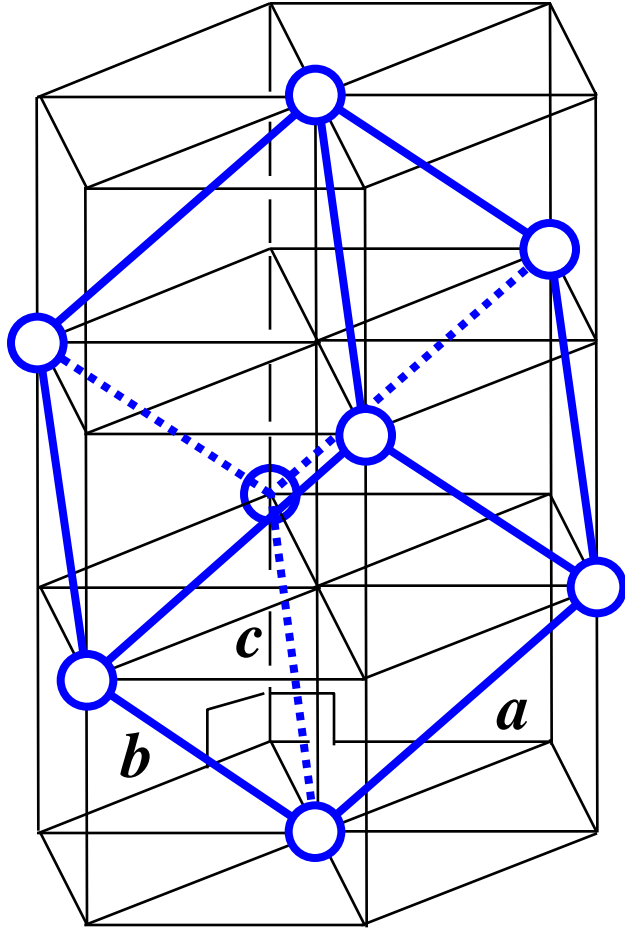


# 晶系 Rhombohedral

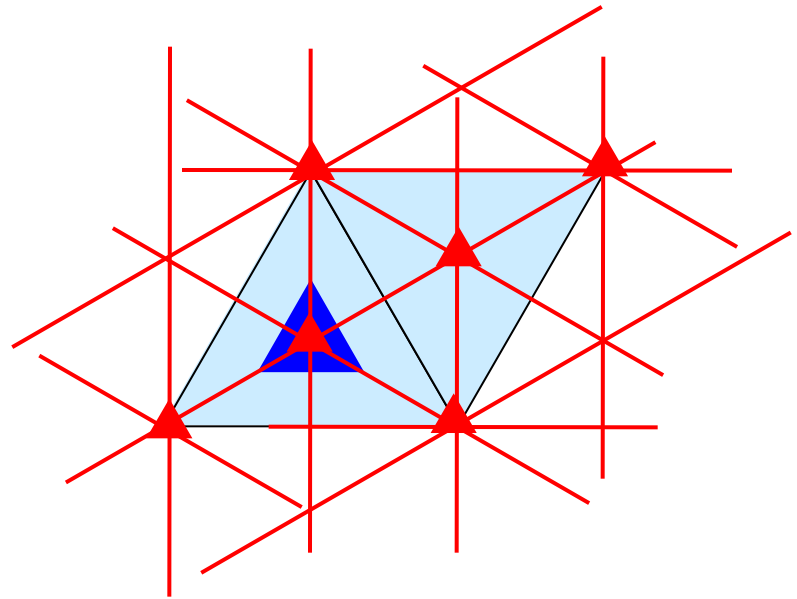
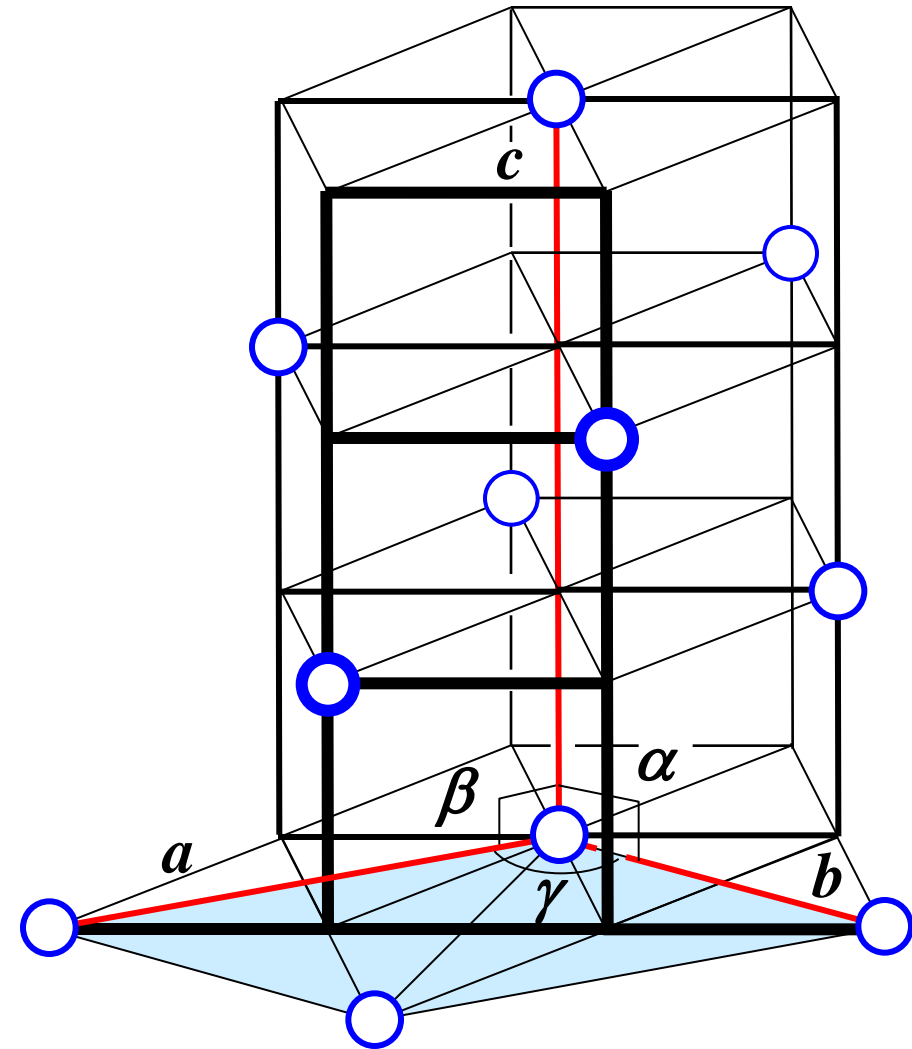




# 晶系 Rhombohedral



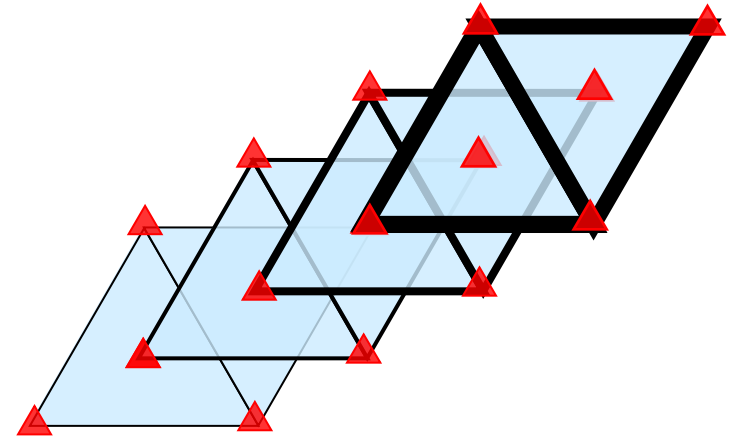
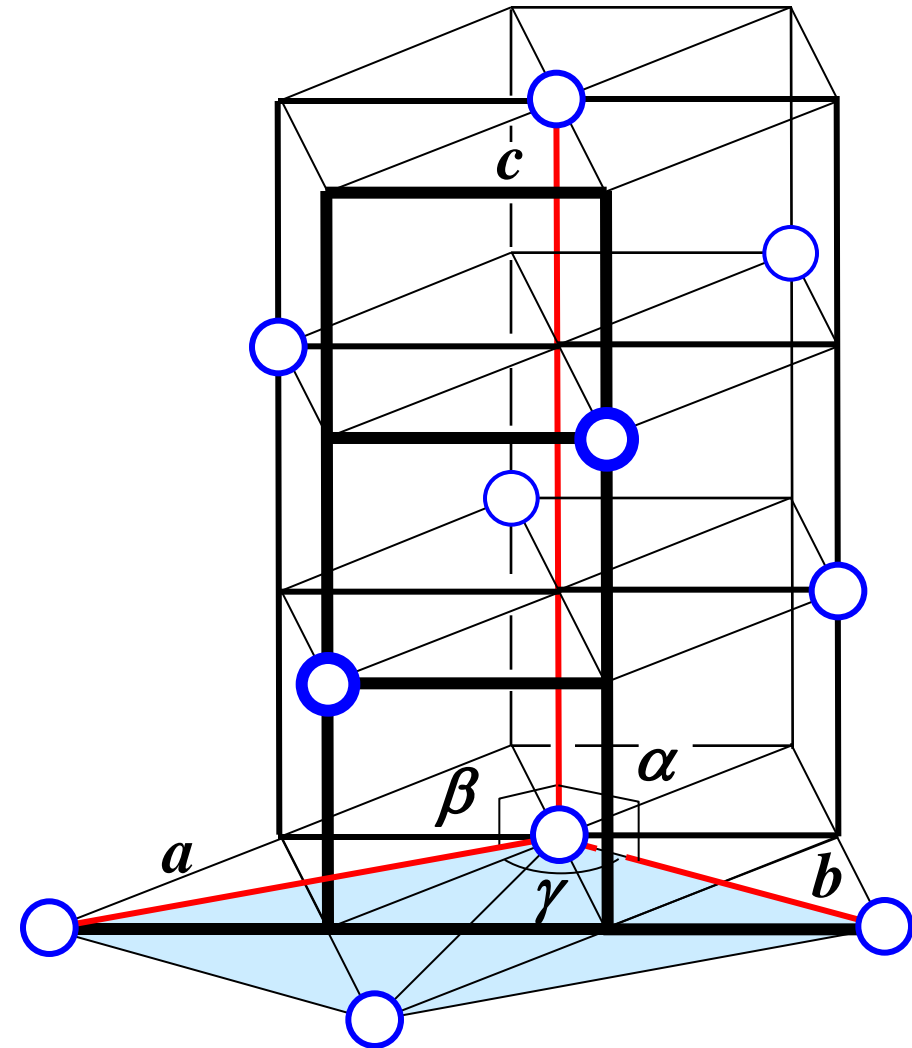
# 晶系 Rhombohedral



$$a = b \neq c,$$

$$\alpha = \beta = 90^\circ, \gamma = 120^\circ$$

# 晶系 Rhombohedral



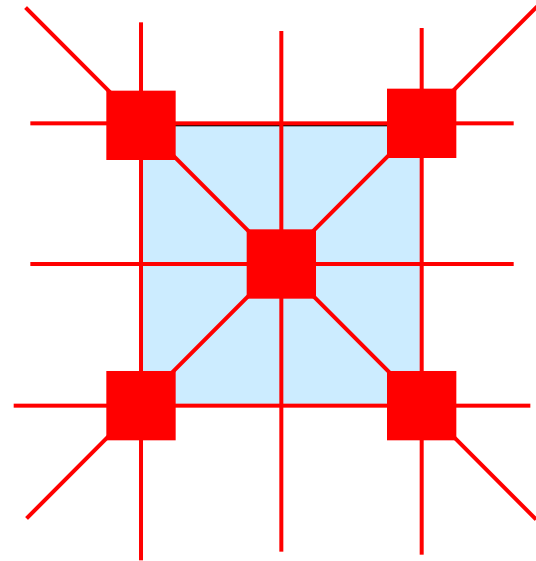
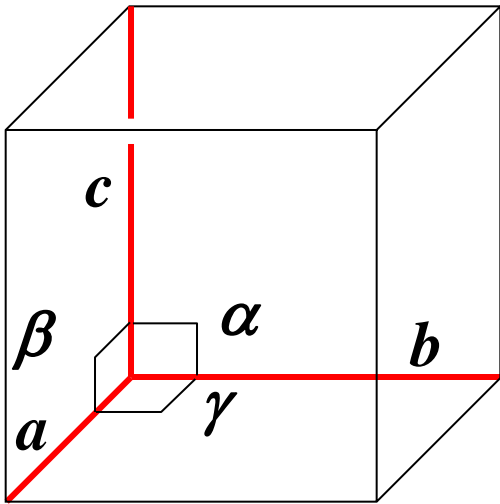
$$a = b \neq c,$$

$$\alpha = \beta = 90^\circ, \gamma = 120^\circ$$

菱面体  $\in$  三方



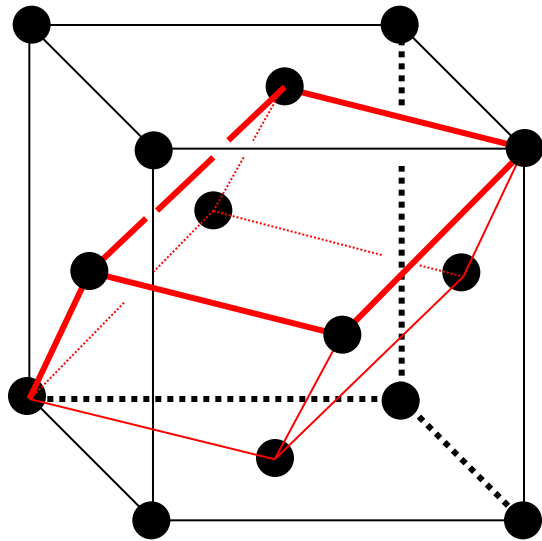
# 晶系 Cubic



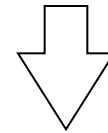
$$a = b = c,$$

$$\alpha = \beta = \gamma = 90^\circ$$

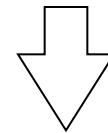
# 複合格子 complex lattice



単純単位胞(赤)は  
結晶の対称性を  
表していない



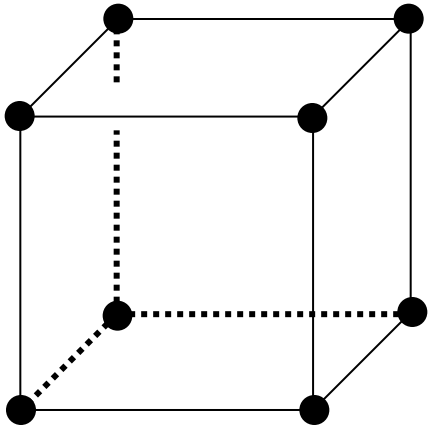
複合格子



Bravais格子

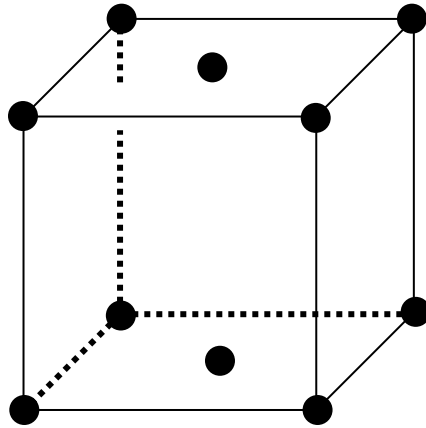


# 複合格子の種類



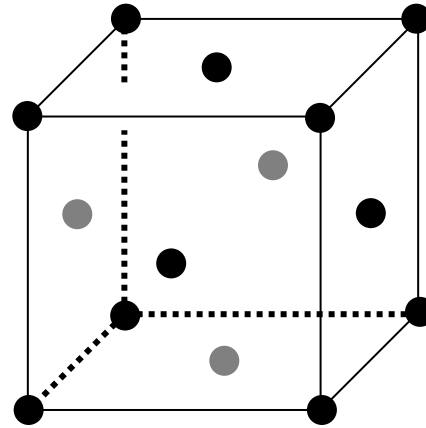
$P$

*primitive*



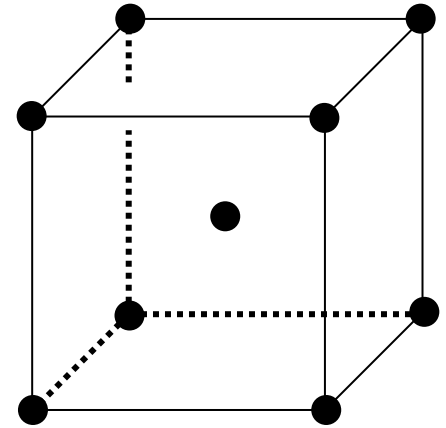
$A, B, C$

*base-centered*



$F$

*face-centered*



$I$

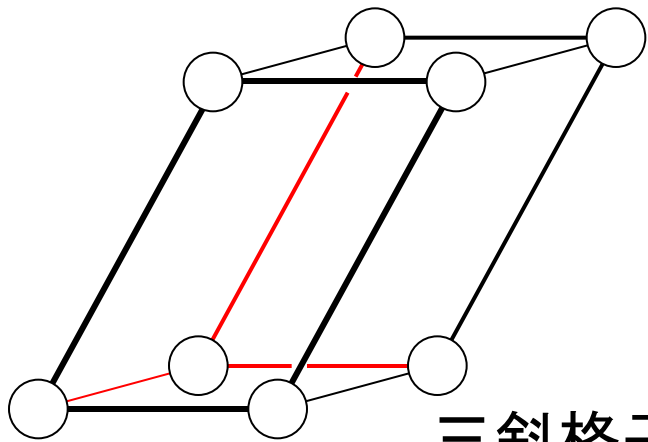
*body-centered*

# Bravais格子

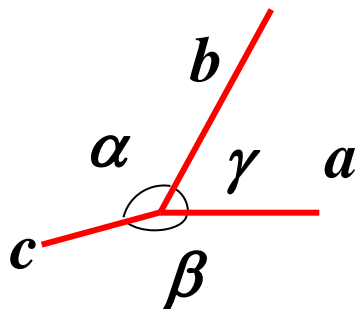
晶系	必須の対称要素	単位胞の形状	Bravais格子
三斜	なし	$a \neq b \neq c, \alpha \neq \beta \neq \gamma$ ( $c < a < b$ )	$P$
単斜	1本の2回軸	$a \neq b \neq c, \alpha = \gamma = 90^\circ, \beta \neq 90^\circ$ ( $c < a$ )	$P, (C, A)$
斜方	3本の2回軸	$a \neq b \neq c, \alpha = \beta = \gamma = 90^\circ$ ( $c < a < b$ )	$P, (C, A, B), I, F$
正方	1本の4回軸	$a = b \neq c, \alpha = \beta = \gamma = 90^\circ$	$P, I$
三方 六方菱面	1本の3回軸	$a = b \neq c, \alpha = \beta = 90^\circ, \gamma = 120^\circ$ $a = b = c, \alpha = \beta = \gamma \neq 90^\circ$	$P$
六方	1本の6回軸	$a = b \neq c, \alpha = \beta = 90^\circ, \gamma = 120^\circ$	$P$
立方	4本の3回軸	$a = b = c, \alpha = \beta = \gamma = 90^\circ$	$P, I, F$

# Bravais格子

## 三斜晶系



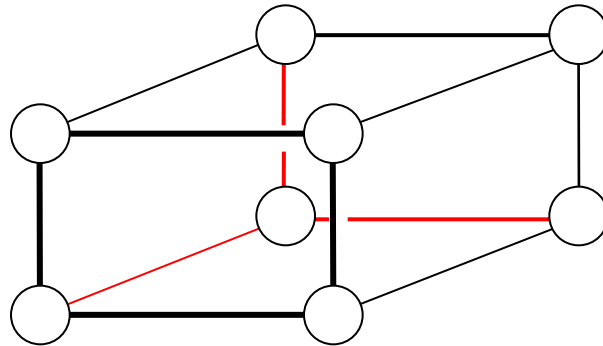
三斜格子



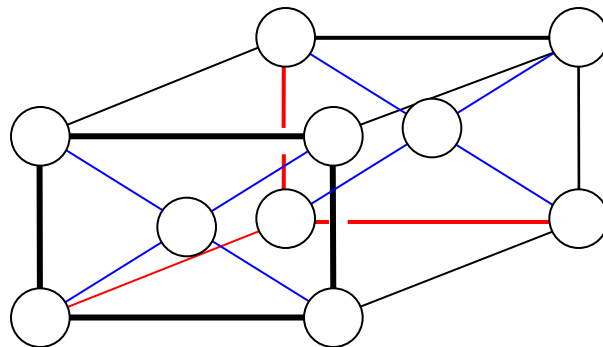
$$a \neq b \neq c, \quad \alpha \neq \beta \neq \gamma$$

$$(c < a < b)$$

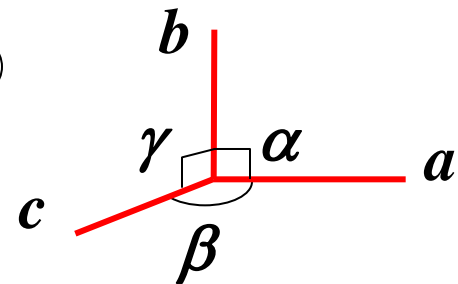
## 单斜晶系



单纯单斜格子



C底心单斜格子  
(A)

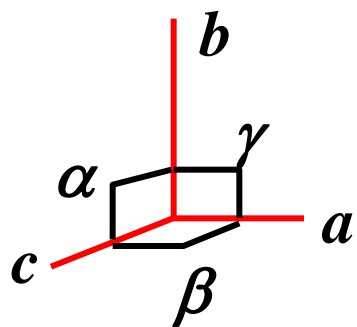


$$a \neq b \neq c, \quad \alpha = \gamma = 90^\circ, \quad \beta \neq 90^\circ$$

$$(c < a)$$

# Bravais格子

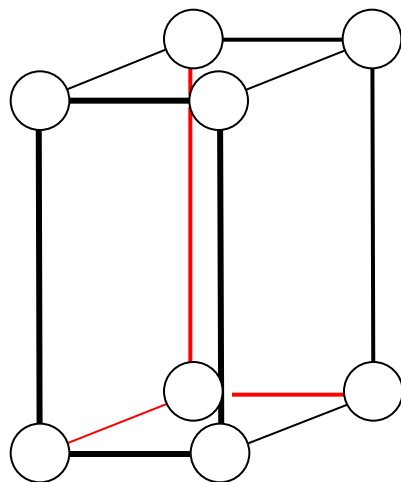
## 斜方晶系



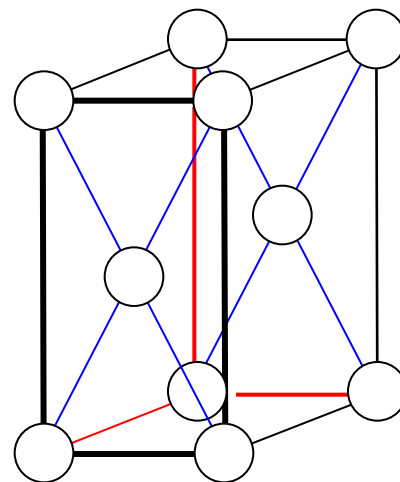
$$a \neq b \neq c,$$

$$\alpha = \beta = \gamma = 90^\circ$$

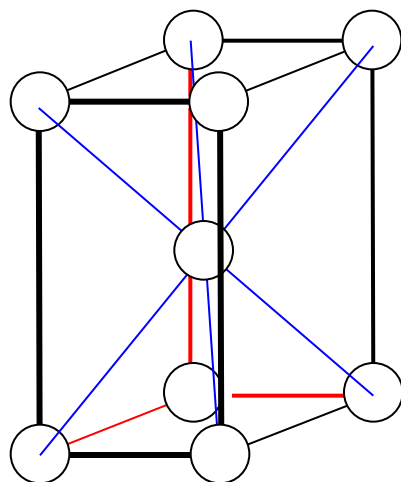
$$(c < a < b)$$



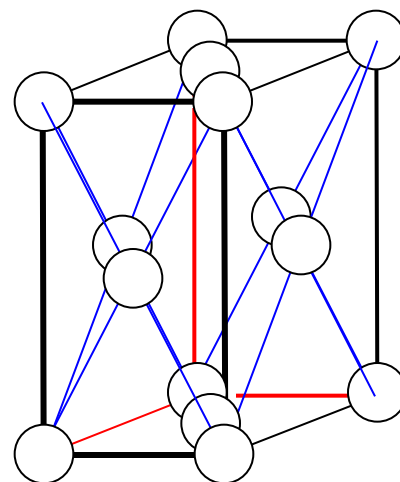
单纯斜方格子



C底心斜方格子(A,B)



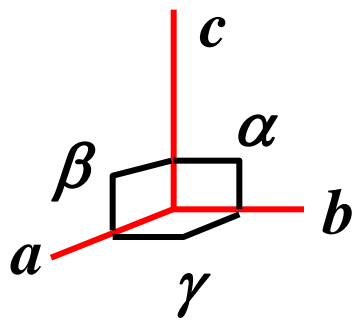
体心斜方格子



面心斜方格子

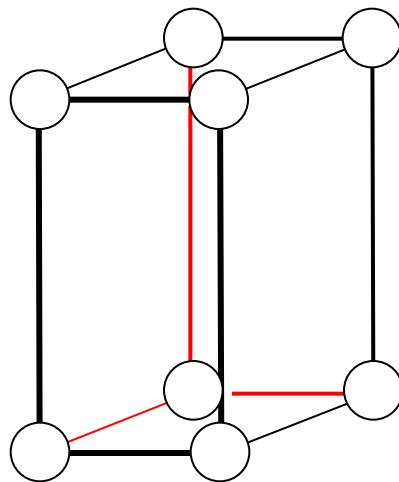
# Bravais格子

## 正方晶系

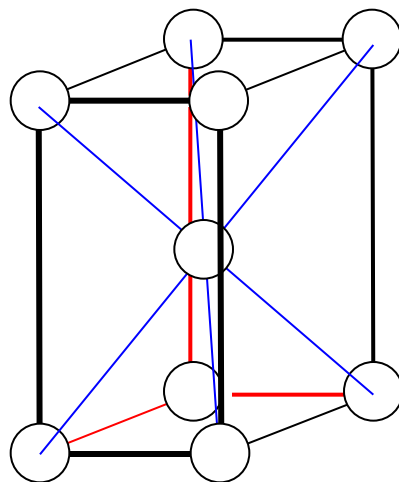


$$a = b \neq c,$$

$$\alpha = \beta = \gamma = 90^\circ$$



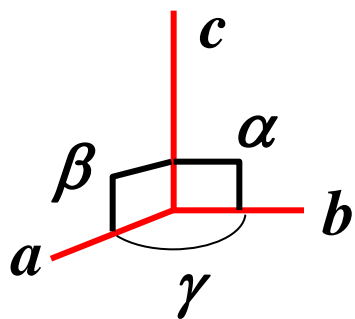
单纯正方格子



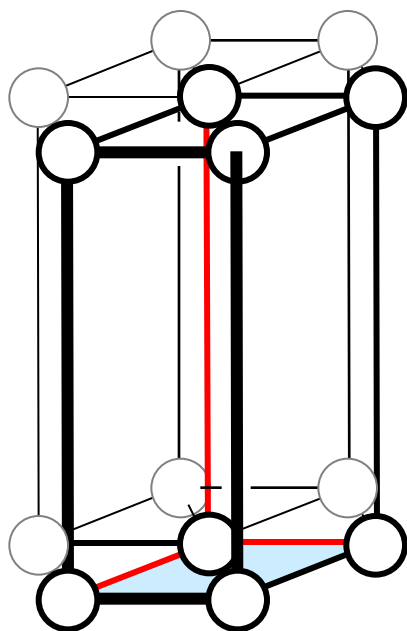
体心正方格子

# Bravais格子

六方晶系  
三方晶系

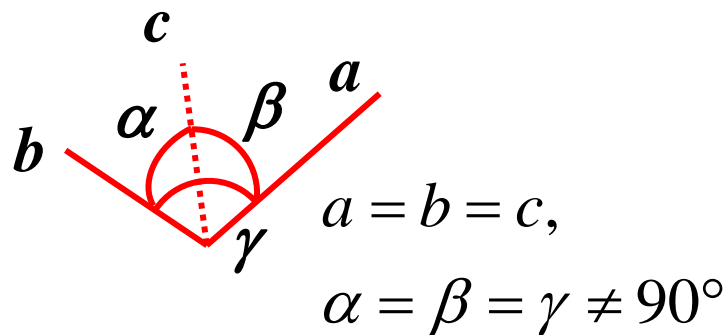


$$a = b \neq c,$$
$$\alpha = \beta = 90^\circ$$
$$\gamma = 120^\circ$$

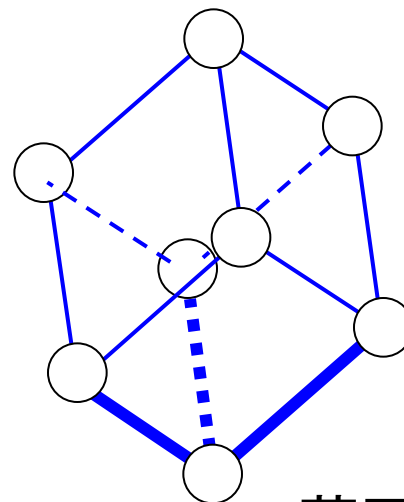


六方格子  
三方格子

菱面体晶系



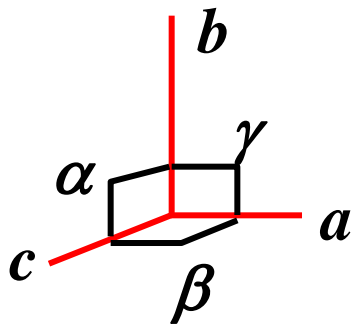
$$a = b = c,$$
$$\alpha = \beta = \gamma \neq 90^\circ$$



菱面体格子

# Bravais格子

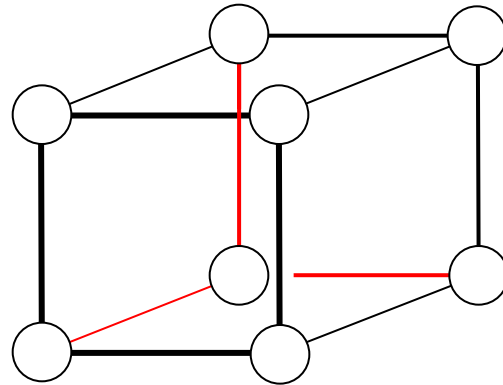
## 立方晶系



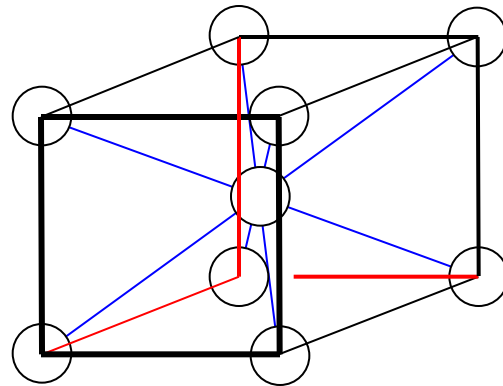
$$a \neq b \neq c,$$

$$\alpha = \beta = \gamma = 90^\circ$$

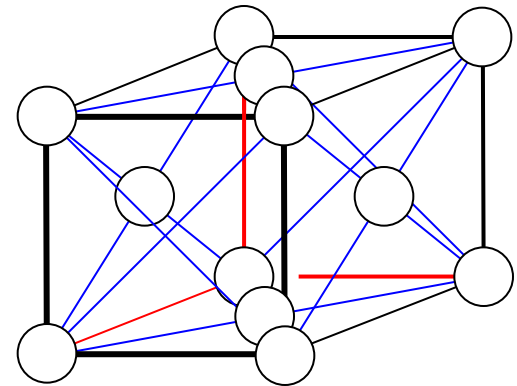
$$(c < a < b)$$



单纯立方格子



体心立方格子



面心立方格子

# まとめ

## ○7つの晶系

三斜、単斜、斜方、正方、三方(菱面体)、  
六方、立方

## ○14種のBravais格子

三斜、単斜( $P, C$ )、斜方( $P, C, I, F$ )、  
正方( $P, I$ )、三方(菱面体)、六方、  
立方( $P, I, F$ )

