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BOOK REVIEW

# Katsuto Tanaka (2017): Time series analysis: nonstationary and noninvertible distribution theory, 2nd edition

## John Wiley & Sons, 889 pp., $135.00, ISBN: 978-1-119-13209-7

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The first edition of Professor Tanaka's book was published in 1996 and has been much appreciated by graduate students and researchers interested in the theoretical underpinnings and computational aspects of what had been "nonstandard limit theory" in the old days. The main theme of the second edition remains the same, but new chapters and sections on more recent developments like unit roots testing under structural change or panel unit root tests have been added. Part II on fractional time series is totally new.

The introductory chapter sets the tone for the book. It discusses examples of statistics related to nonstationarity and noninvertibility of both univariate and multivariate time series. Chapter 2 provides an excellent discussion of topics involved with functional convergence and limiting distributions when (near) unit roots are present in the time series model. It includes an introduction to stochastic calculus and the Itô integral. Then Tanaka turns in three steps to the numerical evaluation of limit distributions involving functionals of Brownian motions. The first approach builds on Girsanov's Theorem and is presented in Chap. 3. A wider class of functionals can be tackled by the second approach contained in Chap. 4 building on Fredholm integral equations. Chapter 5 is devoted to numerical integration, which is required for numerically inverting characteristic functions. The difficulty of dealing with imaginary numbers is addressed, and in fact this chapter may serve as a computational training manual.

The theory and methodology of the previous chapters is then applied to a series of cases in Chaps. 6–11. Chapter 6 treats the estimation of near-integrated autoregressive (AR) models, where the classical near unit root $1 - c/T$ with sample size $T$ is generalized for complex roots and seasonal unit roots, too. Similarly, Chap. 7 addresses the estimation of noninvertible moving average (MA) processes, where a root of the MA polynomial is on or near the unit circle. After discussing estimation, it is natural to approach testing for a unit root, where the case of near-integration or near-invertibility

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allows to provide asymptotic local power functions. A variety of AR unit root tests is dealt with in great detail in Chap. 8, while Chap. 9 is reserved to unit root test in MA models. In this second edition, Chap. 10 is new and copes with a development of the last two decades: panel unit root tests. It assumes a simple panel structure with homogeneity and independence over the units, while the time dimension as well as the cross-sectional dimension are considered to diverge. In this framework, power results differing from the pure time series setting are established for the AR case and novel panel unit root tests are suggested and discussed for the MA case. Chapter 11 contains essentially the same material as in the first edition. It is concerned with the cointegration analysis of multivariate time series, although the starting point is of course the so-called spurious regression theory under absence of cointegration.

Part I of the book ends with Chap. 11, the part on the analysis of non-fractional time series, where the order of integration in an integer, mostly equal to 1. Part II as a whole is new in the second edition and consists of three chapters on fractional time series, where the order of integration $d$ is allowed to be any real (positive) number. Chapter 12 is devoted to the fractional (or fractionally integrated) ARMA model. After discussing stochastic properties, the estimation and testing of $d$ is treated. Next, the fractional Brownian motion (fBm) is introduced as limit of fractional ARMA models just as ARMA processes result in Brownian motions. This chapter also contains a short introduction to spurious regression and cointegration theory under fractional integration. It closes with the estimation of $d$ by using wavelet methods. Chapter 13 provides more details of the fBm. First, it is shown that the methods from Chaps. 3 and 4 are not (directly) applicable to derive the characteristic function of quadratic functionals of fBm. A route to tackle this problem is a martingale approximation to the fBm, which is amenable to the Fredholm approach. Chapter 14 defines a fractional Ornstein–Uhlenbeck process by replacing the Brownian motion by an fBm in its defining equation. Assuming the fractional parameter to be known, estimation of the drift component is discussed, where three cases depending on the sign of the drift parameter have to be dealt with separately. Efficiency of ordinary least squares versus maximum likelihood estimation is investigated and some novel results are proven.

What makes this book special is how proofs are dealt with. Most of them are given in the form of problems whose solutions are presented in Chap. 15 on 160 pages. The relegation of proofs from the main chapters to this final chapter makes the book highly readable while being relatively self-contained at the same time. Graduate students in time series econometrics who are willing and able to accept a challenge will profit a lot from a careful study of the book's theorems and detailed derivations.

The first edition of Professor Tanaka's book has been praised as being worthwhile for students and teachers of advanced asymptotic time series theory as well as for researchers being active in this field. I am convinced that this second edition deserves praise even more.

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