Heterogeneity Foundation of Discretion: Legislation, Execution, and Judgment *

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Abstract

The conventional view of discretion claims informational foundation of discretion; the legislature delegates discretion to the executive office for informational reason. By contrast, the present paper offers heterogeneity foundation of discretion; the more discretion the court has, the more legislators can make their own ideal points realized by executive officers and confirmed by the judge and the closer the final outcomes become to ideal points of some legislators on average. Discretion does good those whom biased policy do bad. This paper constructs a game theoretic model and tests a derived hypothesis by using Japanese legislation data.

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1 Introduction

The conventional view claims informational foundation of discretion; the legislature delegates discretion to the executive office in spite of distributional conflict because the latter has more information about policy than the former (Epstein and O’Halloran, 1999). By contrast, the present paper offers heterogeneity foundation of discretion; even if there is no information gap between the legislature and the executive, legislators have incentive to delegate discretion because the legislature and the executive are composed of not homogeneous but heterogeneous members. The intuitional reason is this; the more discretion the court has, the more legislators can make their own ideal points realized by executive officers and confirmed by the judge and the closer the final outcomes become to ideal points of some legislators on average. Discretion does good those whom biased policy do bad.

In order to explain this, this paper constructs a game theoric model. I derive subgame perfect equilibrium and the following hypothesis: the more biased policy is off the median, the wider discretion is allowed to the court. Though this may not be surprising, I repeat that this does not depend on informational asymmetry between the legislature and the executive. Moreover, I make it clear that discretion is given to the judiciary rather than the executive. The legislature can not directly force the executive to do what it does. It can induce the executive to do what it will do indirectly only through judicial review.

This paper is organized as follows. First, in order to give readers an idea how actors play the game, I illustrate an example of “discrete version,” where ideal points are fixed variables. It is not, however, strict explanation of my model, which I turn to in the following section. There, I present “continuous” version of my model, which represents various ideal points of many legislators by a random variable with distribution. After I explain the general version, a running example follows for illustration. I also derive subgame perfect equilibrium. In the third section, I derive a empirical hypothesis derived from the positive model and test it by
using Japanese legislation data. Finally, I conclude.

2 Example of Discrete Version Model

2.1 Players and Their Ideal Points

2.1.1 Legislators

Consider one dimension policy space. The legislature is composed of two opposition party legislators (O1, O2) and five governing party ones (G1, ..., G5). Their ideal points are $\lambda_{O1} = -5, \lambda_{O2} = -4, \lambda_{G1} = 0, \lambda_{G2} = 1, \lambda_{G3} = 2, \lambda_{G4} = 3, \lambda_{G5} = 3.5$. Let mass probability function of all legislators’ ideal points be $\Lambda(\lambda)$:

$$\Lambda(\lambda) = \begin{cases} 
1/7 & \text{for } \lambda \in \mathcal{L} = \{-5, -4, 0, 1, 2, 3, 3.5\} \\
0 & \text{otherwise}
\end{cases}$$

2.1.2 Executive Officers

I define mass probability function of all executive officers’ ideal points $\varepsilon$’s be $E(\varepsilon)$. I assume that all legislators are also executive officers and implement policy:

$$E(\varepsilon) = \Lambda(\varepsilon).$$

This assumption is not necessary for my model but make it easy to present the argument.

Obviously, this is very strong assumption, while I believe this is not so unreasonable. One reason is parliamentary system. Unlike presidential system, the legislature and the executive have similar ideal points. Another reason is democratic control of bureaucrats. A lawmaker represents a district and, allegedly, its ideal point. If democratic control of bureaucrats is well established, its executive officer will also behave as if it internalized the districts’ ideal
point. The way to discipline bureaucracy varies; lawmakers may force bureaucrats to do what they want by political pressure or inducement such as pork; since the same voters elect national and local representatives, both may reflect the same interest and local politicians have an influence on executive officers through budget or promotion.

2.2 Four Stage Game: Strategies and Outcomes

2.2.1 Legislation of Policy Center: $v_C(\lambda)$ and $C$

A law stipulates center $C$ and width $W$ of policy. Let center of the status quo policy be $C_{SQ} = \lambda_{O2} = -4$. At first, an alternative of policy center $C_{alt} = \lambda_{G4} = 3$ is tabled in the legislature. For some reasons (which I will explain shortly), suppose that $O1$ and $O2$ prefer $C_{SQ}$ but the others choose $C_{alt}$. A strategy of a legislator $\lambda$ is denoted by $v_C(C_{alt}|C_{SQ}, \lambda)$. It takes 1 if the legislator $\lambda$ votes for $C_{alt}$ and 0 if it vote for $C_{SQ}$. In the current example:

$$v_C(C_{alt}|C_{SQ}, \lambda) = \begin{cases} 
1 & \text{for } \lambda = \lambda_{O1}, \lambda_{O2} \\
0 & \text{for } \lambda = \lambda_{G1}, \lambda_{G2}, \lambda_{G3}, \lambda_{G4}, \lambda_{G5}.
\end{cases}$$

Since the legislature makes decision by majority rule, let its decision function of $C$ be

$$C = V_C(C_{alt}|C_{SQ}, \Lambda, v_C) = \begin{cases} 
C_{alt} & \text{if } \frac{1}{N} \sum_{i=1}^{N} v_C(C_{alt}|C_{SQ}, \lambda_i, v_C) > 1/2 \\
C_{SQ} & \text{otherwise}
\end{cases}$$

In the running example, the legislature concludes $C = V_C = C_{alt}$ by 5 to 2.

2.2.2 Legislation of Discretion Width: $v_W(\lambda)$ and $W$

Next, against $W_{SQ} = 2$, an alternative of discretion width $W_{alt} = 1$ is proposed in the legislature. Individual legislator’s strategy $v_W(W_{SQ}|W_{alt}, \lambda, C)$ and the collective legislature’s
decision function \( V_{\text{W}}(W_{\text{SQ}}|W_{\text{alt}}, \Lambda, v_{\text{W}}, C) \) are defined similarly.

\[
v_{\text{W}}(W_{\text{alt}}|W_{\text{SQ}}, \lambda) = \begin{cases} 
1 & \text{if } 1/N \sum_{i=1}^{N} v_{\text{W}}(W_{\text{alt}}|W_{\text{SQ}}, \lambda_i, v_{\text{W}}) > 1/2 \\
0 & \text{otherwise} 
\end{cases}
\]

\[W = V_{\text{W}}(W_{\text{alt}}|W_{\text{SQ}}, \Lambda, v_{\text{W}}) = \begin{cases} 
W_{\text{alt}} & \text{if } 1/N \sum_{i=1}^{N} v_{\text{W}}(W_{\text{alt}}|W_{\text{SQ}}, \lambda_i, v_{\text{W}}) > 1/2 \\
W_{\text{SQ}} & \text{otherwise} 
\end{cases}\]

For some reasons (which, again, I will explain shortly), suppose that O1, O2, G1 and G2 vote for \( W_{\text{SQ}} \) but the others vote for \( W_{\text{alt}} \). In this case, the legislature decides \( W = V_{\text{W}} = W_{\text{SQ}} \) by 4 to 3. Note that legislators winning \( W = W_{\text{SQ}} \) are different from those winning \( C = C_{\text{alt}} \). This implies that discretion is for those whom policy center is not for.

### 2.2.3 Execution of Policy Action: \( i(\varepsilon) \) and \( A(\alpha) \)

At the third stage, an individual executive officer with ideal point \( \varepsilon \) implements an action \( \alpha \) according to strategy \( i(\varepsilon) \):

\[\alpha = i(\varepsilon).\]

Suppose that all executive officers take \( i(\varepsilon) = \varepsilon \). Let mass probability function of all implemented actions be denoted by \( A(\alpha) \). In the running example, \( A(\alpha) = E(\alpha) = \Lambda(\alpha) \).

### 2.2.4 Judgment of the Final Outcomes: \( j(\alpha) \) and \( \Omega(\omega) \)

Finally, the judge decides whether each action \( \alpha \) follows the law or not according to its decision function \( j(\alpha) \); if \( \alpha \) falls in a closed set \( \mathcal{J} = [C - W, C + W] = [1, 5] \), which I call the legal set, the judge will confirm \( \alpha \) as the final outcome \( \omega \); otherwise, the judge will reject
\(\alpha\) and rule that it turns the reserve point \(\alpha_0\) into the final outcome \(\omega\).

\[
\omega = j(\alpha|J, \alpha_0) = \begin{cases} 
\alpha & \text{if } \alpha \in J = [C - W, C + W] \\
\alpha_0 & \text{otherwise.}
\end{cases}
\]

Let \(\alpha_0 = C = 3\) (Even if \(\alpha_0\) is sufficiently away from all players, the following argument does not change substantially). On one hand, since \(\alpha_{G2} = 1, \alpha_{G3} = 2, \alpha_{G4} = 3\) and \(\alpha_{G5} = 3.5\) are in the legal set \([1, 5]\), they are confirmed and become \(\omega_s\). On the other hand, \(\alpha_{O1} = -5, \alpha_{O2} = -4\) and \(\alpha_{G1} = 0\) are rejected and, instead, \(\omega_{O1} = \omega_{O2} = -\omega_{G1} = \alpha_0 = C = 3\) are coerced by the court. Let mass probability function of \(\omega\) be denoted by \(\Omega(\omega)\). In the example, \(\Omega(\omega = 1) = 1/7, \Omega(\omega = 2) = 1/7, \Omega(\omega = 3) = 4/7, \Omega(\omega = 3.5) = 1/7\).

### 2.3 On the Equilibrium Path Behaviors and Outcomes

For expositional purpose, I show an example of on the equilibrium path behavior in the opposite order of game extension. Note that this is not proof of equilibrium, which will come in the next section.

For a player (legislator or executive officer) at \(X\), the utility of a final outcome \(\omega\) is the negative value of distance between both points:

\[
u(\omega|X) = -|\omega - X|.
\]

Of course, a player is supposed to prefer larger utility (closer outcome) to smaller utility (farther one).
2.3.1 Execution of Policy Action: \( i^*(\varepsilon), A^*(\alpha) \) and \( \Omega^* \)

In order to prevent judicial rejection, executive officers \( \varepsilon_{O1} = -5, \varepsilon_{O2} = -4 \) and \( \varepsilon_{G1} = 0 \), whose ideal points are outside of the legal set \( J = [1, 5] \), will implement the boundary point closest to them among the legal set: \( \alpha^*_O1 = \alpha^*_O2 = \alpha^*_G1 = 1 \). Other executive officers implement their own ideal points without worrying about rejection. Given \( C \) and \( W \), the equilibrium strategy \( i^*(\varepsilon) \) is

\[
\alpha^* = i^*(\varepsilon) = \begin{cases}
\varepsilon & \text{if } \varepsilon \in J = [1, 5] \\
\arg \max_{\alpha \in J} -|\alpha - \varepsilon| & \text{if } \varepsilon \notin J \\
1 & \text{if } \varepsilon \leq 1 \\
5 & \text{if } \varepsilon \geq 5
\end{cases}
\]

If \( \varepsilon = 6, \alpha^* = 5 \). As a result, \( A^*(\alpha = 1) = 4/7, A^*(\alpha = 2) = 1/7, A^*(\alpha = 3) = 1/7, A^*(\alpha = 3.5) = 1/7 \). Note that there is a frequent mass on the boundary of the legal set, \( \alpha = 1 \). Moreover, the judge confirms all implementation: \( \Omega^*(\omega) = A^*(\omega) \).

2.3.2 Legislation of Discretion Width: \( v^*_W, V^*_W \) and \( W^* \)

There are seven final outcomes \( \omega^* \)s. When legislature votes on \( W \), legislators are concerned with average utility of them:

\[
u(\Omega^*|X, W) = 1/N \sum_{i=1}^{N} u(\omega^*_i|X, W).
\]

If \( W_{SQ} \) brings about larger average utility for a legislator \( \lambda \) than \( W_{alt} \), the legislator votes for the former. Otherwise, it does not.

When \( \lambda \) is not in the legal set defined by \( W \) (and \( C \)), I call it “outsider”: \( \lambda \notin J(W) \). For this legislator, average utility \( u(\Omega^*|\lambda, W) \) is equal to the negative value of distance between
the legislator and average final outcome, $\bar{\omega}^* = 1/N \sum_{i=1}^{N} \omega^*$:

$$u(\Omega^*|\lambda \notin J(W), W) = -|\bar{\omega}^* - \lambda|.$$ 

For example, $\lambda_{O1} = -5$ is an outsider of the legal set $J(W_{SQ}) = [3 - 2, 3 + 2]$ and of $J(W_{alt}) = [3 - 1, 3 + 1]$. Then,

$$u(\Omega^*|\lambda_{O1}, W_{SQ} = 2) = -(3 \times 0 + 2 + 3 \times 0 + 3.5)/7 = -(6 + 5.5/7)$$

$$u(\Omega^*|\lambda_{O1}, W_{alt} = 1) = -((2 \times 5 + 3 \times 3 + 3.5)/7 + 5) = -(7 + 2.5/7)$$

Thus, O1 prefers $W_{SQ}$. Note that average outcome is not necessarily located on policy center $C$.

When $\lambda$ is in the legal set, I call it “insider”. For this legislator, $u(\Omega^*|\lambda, W)$ is not equal to the negative value of distance between the legislator and average final outcome. For example, $\lambda_{G4} = 3$ is an insider of the legal sets $J(W_{SQ})$ and $J(W_{alt})$.

$$u(\Omega^*|\lambda_{G4}, W_{SQ} = 2) = -(|1 - 3| \times 4 + |2 - 3| + |3 - 3| + |3.5 - 3|)/7 = -9.5/7$$

$$u(\Omega^*|\lambda_{G4}, W_{alt} = 1) = -(|2 - 3| \times 3 + |3 - 3| + |3.5 - 3|)/7 = -5.5/7$$

Thus, this legislator likes $W_{alt}$ than $W_{SQ}$.

In this way, we know that, in equilibrium, O1, O2, G1 and G2 vote for $W_{SQ}$, though G3, G4 and G5 vote for $W_{alt}$. Accordingly, the equilibrium strategy of individual legislator is

$$v^*_W(W_{SQ}|W_{alt}, \lambda) = \begin{cases} 
1 & \text{for } u(\Omega^*|\lambda, W_{SQ}) > u(\Omega^*|\lambda, W_{alt}) \quad \text{(i.e. } \lambda = \lambda_{O1}, \lambda_{O2}, \lambda_{G1}, \lambda_{G2}) \\
0 & \text{otherwise.}
\end{cases}$$

Thus, $W^* = V^*_W = W_{SQ}$. 

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2.3.3 Legislation of Policy Center: \( v^*_C, V^*_C \) and \( C^* \)

If \( C = C_{SQ} = \lambda_{o2} = -4 \), \( W^*(C_{SQ}) = 5 \) for the reason I will explain later. Then, the legal set \( \mathcal{J}^*(C_{SQ}) = [C_{SQ} - W^*(C_{SQ}), C_{SQ} + W^*(C_{SQ})] = [-9, 1] \). In the case of \( C = C_{alt} = 3 \), as I show above, \( W^*(C_{alt}) = 2 \) and the legal set \( \mathcal{J}^*(C_{alt}) = [1, 5] \). Utility is calculated for insiders and outsiders and the equilibrium strategy of individual legislator, \( v^*_C(C_{alt}|C_{SQ}, \lambda) \), is defined as explained in the previous paragraphs. In equilibrium, O1, O2 and G1 choose \( C_{SQ} \), though G2, G3, G4 and G5 select \( C_{alt} \). Thus, \( C^* = V^*_C = C_{alt} \) results.

3 Continuous Version Model

3.1 Players and Their Ideal Points

3.1.1 Legislators: \( \lambda \sim \Lambda(\lambda) \)

General Version. Thinking that there are sufficiently large number of legislators, I approximate their collective behavior by random variable. I assume that any legislator’s ideal point \( \lambda \) follows the distribution \( \Lambda(\lambda) \) (all mass probability functions in the previous section read as mixed distribution of mass probability function and probability density). This means that the share of legislators whose ideal points are from \( \lambda_{min} \) to \( \lambda_{max} \) is \( \int_{\lambda_{min}}^{\lambda_{max}} \Lambda(\lambda)d\lambda \).

Example. As a running illustrative example of continuous version model, I use the following case for computational reason. The governing party and the opposition party occupy \( 0.5 + M \) and \( 0.5 - M \) of seats, respectively \( (0 < M < 1/2) \). Policy difference between the two parties is \( D \geq 0 \). Then, the following distribution of legislators is one which represents this situation:

\[
\Lambda(\lambda) = \begin{cases} 
1 & \text{if } \lambda \in \mathcal{L} = [-0.5 - M, -D] \cup [0, 0.5 + M] \\
0 & \text{otherwise.}
\end{cases}
\]
The governing party has legislators in the policy space between 0 and 0.5 + \(M\). Within it, members are distributed uniformly. Similarly, the opposition party has legislators whose ideal points are located between \(-0.5 - M\) and \(-D\). Within it, again, intervals between spatially consecutive lawmakers are constant. Moreover, the most right member of the opposition party stands at the distance of \(D\) left from the most left member of the governing party. \(M\) and \(D\) represents seat share margin and policy difference between the two parties, respectively.

3.1.2 Executive Officers: \(\varepsilon \sim E(\varepsilon)\)

**General Version.** I assume that any executive officer’s ideal point \(\varepsilon\) follows the distribution \(E(\varepsilon)\).

**Example.** I assume that the distribution of executive officers is the same as that of legislators: \(E(\varepsilon) = \Lambda(\varepsilon)\).

3.2 Four Stage Game: Strategies and Outcomes

3.2.1 Legislation of Policy Center: \(C = V_C(C_{alt}|C_{SQ}, \Lambda(\lambda), v_C)\)

The status quo center \(C_{SQ} \in \mathbb{R}\) is given. Suppose that the agenda setter whose ideal point is \(S\) proposes an alternative center \(C_{alt}\). Then, all legislators vote on \(C_{SQ}\) and \(C_{alt}\). The legislature decides \(C = V_C(C_{alt}|C_{SQ}, \Lambda(\lambda), v_C)\).

3.2.2 Legislation of Discretion Width: \(W = V_W(W_{alt}|W_{SQ}, \Lambda(\lambda), v_W, C)\)

The status quo discretion \(W_{SQ} \geq 0\) is given. Nature decides an alternative discretion \(W_{alt}\) by chance. Then, all legislators vote on \(W_{SQ}\) and \(W_{alt}\). The legislature resolves \(W = V_W(W_{alt}|W_{SQ}, \Lambda(\lambda), v_W, C)\).
3.2.3 Execution of Policy Action: \( \alpha = i(\varepsilon) \sim A(\alpha|E(\varepsilon), i(\varepsilon)) \)

Since executive officer’s ideal point \( \varepsilon \) is a random variable, so is the action it implements \( \alpha = i(\varepsilon) \) whose distribution is denoted by \( A(\alpha) \). This shows how many executive officers implemented what actions. If the implementation strategy function \( i(\varepsilon) \) has its inverse function \( i^{-1}(\alpha) \), using Jacobian,

\[
A(\alpha) = E(i^{-1}(\alpha)) \left| \frac{\partial i^{-1}(\alpha)}{\partial \alpha} \right|
\]

3.2.4 Judgment of the Final Outcome: \( \omega = j(\alpha) \sim \Omega(\omega|C, W, \alpha_0, A(\alpha)) \)

Since \( \alpha \) is a random variable, so is the final outcome it leads to, \( \omega = j(\alpha) \), whose distribution is denoted by \( \Omega(\omega) \). This shows how often every final outcome is realized by the judge.

\[
\Omega(\omega|J = [C - W, C + W], \alpha_0, A(\alpha)) = \begin{cases} 
A(\omega) & \text{if } \omega \in J, \omega \neq \alpha_0 \\
\int_{\omega \notin J} A(\omega)d\omega & \text{if } \omega = \alpha_0 \\
0 & \text{if } \omega \notin J
\end{cases}
\]

3.3 Equilibrium

This four stage game is a dynamic game of complete and perfect information. You might think that this is a game of incomplete information: a legislator and an executive officer don’t know each other’s ideal points but have belief about them \( (E \) and \( \Lambda ) \). You can take the game that way, though that is not the only interpretation. Mine is that there are sufficiently large number of legislators and executive officers and, thus, their collective behavior can be approximated by random variable. Therefore, equilibrium should be sub-game perfect, which is derived by backward induction.
Since $\omega$ is randomly distributed, a player $X$ is concerned with expected utility:

$$u(\omega|X) = \int -|\omega - X|\Omega(\omega)d\omega.$$ 

If an alternative $\omega_{alt}$ has the same utility as that of the status quo $\omega_{SQ}$, $u(\omega_{alt}|X) = u(\omega_{SQ}|X)$, a player is supposed to prefer the status quo.

### 3.3.1 Execution of Policy Action: $i^*(\varepsilon)$, $A^*(\alpha^*)$ and $\Omega^*(\omega^*)$

**General Version.** Given the court’s decision function $j(\alpha)$, what implementation $\alpha$ is the best response for an executive officer $\varepsilon$? If the ideal point of an executive officer is in the legal set $\mathcal{J} = [C - W, C + W]$ and it is implemented, the court will confirm it. This is the best for the executive officer. If the ideal point is out of the legal set, the executive officer will implement the closest point in the legal set and the court will confirm it. Thus, the equilibrium strategy $i^*(\varepsilon)$ is

$$\alpha^* = i^*(\varepsilon) = \begin{cases} 
\varepsilon & \text{if } \varepsilon \in \mathcal{J} = [C - W, C + W] \\
\arg\max_{\alpha \in \mathcal{J}} -|\alpha - \varepsilon| & \text{if } \varepsilon \notin \mathcal{J}
\end{cases}$$

When all the executive officers take strategy $i^*(\varepsilon)$, the distribution of all the equilibrium implemented actions is represented by

$$A^*(\alpha^*) = \begin{cases} 
E(i^{*-1}(\alpha^*)) \left| \frac{\partial i^{*-1}(\alpha^*)}{\partial \alpha^*} \right| = E(\alpha^*) & \text{if } \alpha^* \in (C - W, C + W) \\
\int_{-\infty}^{C-W} E(\alpha^*) d\alpha^* & \text{if } \alpha^* = C - W \\
\int_{C+W}^{\infty} E(\alpha^*) d\alpha^* & \text{if } \alpha^* = C + W \\
0 & \notin \mathcal{J} = [C - W, C + W]
\end{cases}$$
In equilibrium, the judge will confirm all implemented actions and the final outcome $\omega^*$ will follow $\Omega^*(\omega^*) = A^*(\omega^*)$.

Example. The equilibrium individual strategy is

$$\alpha^* = i^*(\varepsilon) = \begin{cases} 
C - W & \text{if } \varepsilon < C - W \\
\varepsilon & \text{if } C - W < \varepsilon < C + W \\
C + W & \text{if } \varepsilon > C + W 
\end{cases}$$

The distribution of the equilibrium implemented action is

$$A^*(\alpha^*) = \begin{cases} 
\int_{-\infty}^{C-W} \Lambda(\alpha^*) d\alpha^* & \text{if } \alpha^* = C - W \\
\Lambda(\alpha^*) = 1 & \text{if } C - W < \alpha^* < C + W, \quad \alpha^* \in \mathcal{L} \\
\Lambda(\alpha^*) = 0 & \text{if } C - W < \alpha^* < C + W, \quad \alpha^* \notin \mathcal{L} \\
\int_{C+W}^{\infty} \Lambda(\alpha^*) d\alpha^* & \text{if } \alpha^* = C + W \\
0 & \text{if } \alpha^* < C - W \text{ or } \alpha^* > C + W 
\end{cases}$$

where

$$\int_{-\infty}^{X} \Lambda(\alpha^*) d\alpha^* = \begin{cases} 
0 & \text{if } X \leq -(0.5 - M) - D \\
X + (0.5 - M) + D & \text{if } -(0.5 - M) - D \leq X \leq -D \\
0.5 - M & \text{if } -D \leq X \leq 0 \\
X + 0.5 - M & \text{if } 0 \leq X \leq 0.5 + M \\
1 & \text{if } 0.5 + M \leq X 
\end{cases}$$

$$\int_{-\infty}^{\infty} E(\alpha^*) d\alpha^* = 1 - \int_{-\infty}^{X} E(\alpha^*) d\alpha^*$$
### 3.3.2 Legislation of Discretion Width: $W^* = V^*_W$

**General Version.** Let $\mathcal{W}(W_{SQ})$ be the set of alternative discretion $W_{alt}$’s which majority legislators prefer to $W_{SQ}$ (i.e. the winset of $W_{SQ}$):

$$\mathcal{W}(W_{SQ}) = \{W | V_W(W|W_{SQ}) = W \}$$

If $\mathcal{W}(W^*_{SQ}) = \emptyset$, the legislature passes $W^*_{SQ}$ and $W^* = V^*_W = W^*_{SQ}$. Otherwise, an alternative $W_{alt} \in \mathcal{W}(W_{SQ})$ is proposed and passed: $W^* = V^*_W = W_{alt}$.

**Example.** $E(\varepsilon) = \Lambda(\varepsilon)$ is divided uniform distribution. An insider legislator $\lambda \in \{\lambda | W \geq |\lambda - C|\}$ prefers narrower discretion $W$ for the fixed policy center $C$, because more executive officers $\varepsilon \in \{\varepsilon | W \leq |\varepsilon - C|\}$ implement actions closer to legislators (at $C - W$ or $C + W$).

For an outsider legislator $\lambda \not\in J$, its utility is the negative value of its distance from the expected final outcome $\bar{\omega}^*$:

$$\bar{\omega}^* = \int \omega^* \Omega^*(\omega^*) d\omega^*$$

$$u(\Omega^*|\lambda \not\in J) = -|\bar{\omega}^* - \lambda|$$

Suppose $J = [C - W, C + W] \subseteq [0, 0.5 + M]$. In equilibrium, executive officers $\varepsilon \geq C + W$ whose share is $(0.5 + M) - (C + W)$ implement $a^*(\varepsilon) = C + W$, executive officers $\varepsilon \leq C - W$ whose share is $(0.5 - M) + (C - W)$ implement $a^*(\varepsilon) = C - W$, and executive officers $C - W \leq \varepsilon \geq C + W$ whose share is $2W$ implement $a^*(\varepsilon) = \varepsilon$. All of them are confirmed.
by the judge and become the final outcomes \( \omega^* \). Then,

\[
\bar{\omega}^* = \int \omega^* \Omega^*(\omega^*) d\omega^*
\]

\[
= ((0.5 + M) - (C + W)) \times (C + W) + ((0.5 - M) + (C - W)) \times (C - W) + 2W \times C
\]

\[
= C + 2(M - C)W
\]

\[
= (1 - 2W) \times C + 2W \times M
\]

This means that, as discretion \( W \) becomes larger, the expected final outcome \( \bar{\omega}^* \) in equilibrium is closer to the median \( M \) than the policy center \( C \) is. That is, discretion corrects biased legislation towards the median. If \( C \leq M \), I call legislator \( \lambda \geq C + W \) majority outsider and \( \lambda \leq C - W \) minority outsider. If \( C \geq M \), these labels are exchanged. Then, for the fixed policy center \( C \), wider discretion brings the expected final outcome \( \bar{\omega}^* \) closer to \( M \) and majority outsiders but farther away from \( C \) and minority outsiders.

From the above argument, for the fixed policy center \( C \),

- As long as an legislator is an insider, it prefers narrower discretion.
- As long as an legislator is a majority outsider, it prefers wider discretion.
- As long as an legislator is a minority outsider, it prefers narrower discretion.

These are true even if \( \mathcal{J} = [C - W, C + W] \subseteq [0, 0.5 + M] \) does not hold.

Let \( W_{SQ}^* = |C - M| \). Below, I show that its winset is empty. When \( 0 \leq W_{alt} < M - C \), more than half legislators \( \lambda \geq (\leq)M \) vote for the larger discretion \( W_{SQ}^* \), because they are majority outsiders. What if \( 0 \leq M - C < W_{alt} \)? Since legislators \( \lambda \in \{ \lambda | C - W_{SQ}^* \leq \lambda \leq M \} \) are insiders of both legal sets \( (\mathcal{J} (W_{alt}) \text{ and } \mathcal{J} (W_{SQ}^*)) \), they prefer the smaller discretion, \( W_{SQ}^* \). Since legislators \( \lambda \in \{ \lambda | \lambda \leq C - W_{alt} \} \) are minority outsiders of both legal sets, they
also prefer the smaller discretion, $W_{SQ}^*$. For legislators $\lambda \in \{\lambda | C - W_{alt} < \lambda < C - W_{SQ}^*\}$,

$$u(\Omega^*|\lambda, W = W_{SQ}^*) > u(\Omega^*|\lambda, W_{SQ}^* < W = |\lambda - C| < W_{SQ}^*) \quad (\because \text{ minority outsider})$$

$$> u(\Omega^*|\lambda, W = W_{alt}) \quad (\because \text{ insider})$$

Thus, more than half legislators $\lambda \leq M$ vote for $W_{SQ}^*$. The case of $M > C$ is similar. Therefore, whatever $W_{alt}$ Nature chooses, majority legislators prefer $W_{SQ}^*$ and its winset is empty. From the above, $W_{SQ}^*$ will be the equilibrium discretion size $W^*$.

### 3.3.3 Legislation of Policy Center: $C^* = V_C^*$

**General Version.** Let $\mathcal{C}(C_{SQ})$ be the winset of $C_{SQ}$:

$$\mathcal{C}(C_{SQ}) = \{C | V_C(C|C_{SQ}) = C\}$$

In equilibrium, the agenda setter $S$ will propose the closest point in this winset as the alternative discretion $C_{alt}^*$:

$$C_{alt}^* = \arg \max_{C \in \mathcal{C}(C_{SQ})} u(\Omega^*|\lambda = S, C)$$

And this is passed by the legislature: $C^* = V_C^* = C_{alt}^*$.

**Example.** When $M < C$, the equilibrium legal set will be $\mathcal{J}^*(C) = [C - W^*(C), C + W^*(C)] = [M, 2C - M]$. When $M > C$, it will be $\mathcal{J}^*(C) = [2C - M, M]$. Suppose that the agenda setter is located in the right side of the median: $S \geq M$. For it, $C_{alt}^* = (S + M)/2$ is best.

When $C_{alt}^* \leq C_{SQ}$, more than half legislators $\lambda \leq M$ are majority outsiders of both legal sets and prefer $C_{alt}^*$ because $\bar{w}^*(C_{alt}^*) < \bar{w}^*(C_{SQ})$. Thus, the agenda setter proposes $C_{alt}^*$ and the legislature passes it: $C^* = C_{alt}^*$. 

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When $M \leq C_{SQ} \leq C^*_{alt}$, the same legislators $\lambda \leq M$ prefer $C_{SQ}$ to any $C_{alt} \geq C_{SQ}$. The agenda setter is an outsider of the legal sets and the expected final outcome $\bar{\omega}^*(C)$ is closer to it as $C$ is larger. Thus, the agenda setter proposes $C^*_{SQ}$ and the legislature passes it: $C^* = C^*_{SQ}$.

What if $C_{SQ} \leq M$? Let reverse point $C^{**}_{alt}$ be the policy center whose equilibrium legal set brings about the same utility to the median as the status quo policy center and which is located in the other half side as to $M$ from the status quo policy center: $M - \bar{\omega}^*(C_{SQ}) = \bar{\omega}^*(C_{alt}^{**}) - M$. When $C_{SQ} \geq M/2$, $C_{alt}^{**} = 2M - C_{SQ} > M$. When $C_{SQ} < M/2$, $C_{alt}^{**} > 2M - C_{SQ} > M$. If $C_{alt}^{**} < C_{alt}^*$, $S$ will propose $C_{alt}^*$ and more than half legislators $\lambda \geq M$ vote for it: $C^* = C_{alt}^*$. Otherwise, $S$ will propose $C_{alt}^*$ and the same legislators $\lambda \geq M$ vote for it: $C^* = C_{alt}^*$.

The case of $S \leq M$ is a mirror image of the above argument.

3.3.4 Summary

In the running example, on the equilibrium path, each player produces the following outcomes.

**Legislation of Policy Center.** The agenda setter $S$ makes the legislature pass policy center $C^*$ biased in favor of itself as much as possible.

**Legislation of Discretion Width.** The legislature sets discretion $W^* = |C^* - M|$ so that the median $M$ and the agenda setter $S$ are on the boundaries of the legal set $\mathcal{J}$, $C^* - W^*$ and $C^* + W^*$, and corrects bias of policy $C$ in favor of the median $M$ to some degree.

**Execution of Action.** Executive officers implement their ideal points as long as they are in the legal set. Otherwise, they execute the closet point in the legal set.

**Judgment of the Final Outcome.** The judge confirms all implementation: $j(\alpha^*) = \alpha^*$. 

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4 Statistical Test

The most important implication of my model is that discretion is the distance between the policy center and the median legislator; \( W^* = |C^* - M| \). Accordingly, the following hypothesis is derived.

**Hypotheses:** The farther policy center \( C \) is away from the median \( M \), the larger discretion \( W \) is.

I will test this by using data of Japanese government bills which are passed by a committee with a resolution in the House of the Representatives from 1978 to 2000 (N=1202). I suppose that \( C^* \) is increasing in \( Party \), the number of relevant parties (Sartori, 1976) which oppose a bill. Thus, I measure \( |C^* - M| \) by a quadratic function of Party. Since there are five relevant parties during this period, Party takes from zero to five.

I measure \( W \) by *Discretion*, the negative number of sentences in committee resolution attached to a bill (I also measure it by the number of items, which are composed of one sentence or a few sentences, though the conclusion does not change). Committee resolution is attached to an original bill to make clear its interpretation or application. Therefore, the more words are spent, the more precise what the judge confirms is and the narrower discretion is. This idea follows Huber and Shipan (2002) and Martin (2004). Discretion’s mean is 7.4 and standard deviance is 4.1.

Then, I regress Discretion on Party and its square;

\[
Discretion = \beta_0 + \beta_1 Party + \beta_2 Party^2 + \epsilon
\]

where \( \epsilon \) is an error term. Since the dependent variable is count, I use negative binomial regression and parameterize logarithm of expectation by linear predictor. My prediction is \( \beta_1 > 0, \beta_2 < 0, \min(\text{Party}) < \text{Party}_M = \frac{-\beta_1}{2\beta_2} < \max(\text{Party}) \).
Table 1 shows the result, which supports the present model. All coefficients are significantly different from zero at 1% level. As I anticipated, $\beta_1 = 0.250 > 0, \beta_2 = -0.064 < 0, \min(Party) = 0 < Party_M = \frac{-\beta_1}{2\beta_2} = 1.941 < \max(Party) = 5$. When too many or too few parties vote against a bill, namely, policy center $C$ is away from the median $M$ very much, committees attach simpler resolution to admit discretion for executive officers and the judge so that the final outcomes become closer to the median and farther away from biased policy center $C$ than it would be without discretion.

<table>
<thead>
<tr>
<th></th>
<th>Coef</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Party</td>
<td>0.250</td>
<td>0.037 **</td>
</tr>
<tr>
<td>Party$^2$</td>
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<td>0.009 **</td>
</tr>
<tr>
<td>Constant</td>
<td>1.918</td>
<td>0.023 **</td>
</tr>
<tr>
<td>log(Dispersion)</td>
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<td>0.080 -</td>
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</tbody>
</table>

Table 1: Result of Negative Binomial Regression

5 Conclusion

Why discretion? Because majority (outsider) legislators would like to make executive officers implement and the judge confirm policy which a law does not intend exactly. The main argument is that the legislature is composed of heterogeneous members and not all is completely happy with a law’s policy (center, $C$) itself.

This model also presents other interesting implication, though I do not derive explicit hypothesis. In the first place, what is legislation? My model supposes that a law decides not the exact location of every implementation ($C$) but just its distribution shape ($W$). Unintuitively, impartial judgment of the court creates lopsided outcomes because the executive officers are asymmetrically distributed.

Future research agendas are in order. This model assumes that the judge is not concerned with its own ideal point and has deterministic decision function. Promising extension is a
game where the court is composed of several judges with their ideal points and has probabilistic judgment strategy. In addition, the reservation point may be not $C$ but other points such as status quo, judge’s ideal point or where (they think) the Constitution stands.

This model supposes that, within each party, lawmakers’ ideal points are uniformly distributed. It would be preferable to assume that members are more centripetal.

I do not deny informational foundation of discretion, though I do not agree that it is the only reason why the legislature admits discretion to the executive through court’s judgment. This paper aims to establish heterogeneity foundation of discretion.
References


