Why Do Bicameral Chambers Usually, but Not Always, Agree? An Incomplete Information Game Model *

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Abstract

Naive observers wonder why the bicameral conference is not held after the second chamber's amendment of the first chamber's bill, while complete information models fail to explain why the conference is sometimes held. This paper addresses both questions by constructing an incomplete information model. The more uncertain a chamber is of the other's position or the more important a bill is, the more likely the bill is to be amended or taken to the conference.

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INTRODUCTION

Why the bicameral conference, though rarely, but does, occur? The constitutions prescribe the bicameral conference as reconciliation tool for bicameral conflict. In reality, even though the second chamber amends a bill from the first chamber and sends it back to the first chamber, the first chamber accepts it as it is and rarely requests the bicameral conference.

Complete information models argue that, since the second chamber amends a bill so that the first chamber accepts it, the first chamber does not have to appeal to the bicameral conference. They even predict no occurrence of the bicameral conference. The problem is, however, that the bicameral conference is sometimes called. Moreover, thess models necessitate frequent amendment by the second chamber, while it is not the case in real lawmaking.

The legislative literature fails to explain *both* why the bicameral conference is not so often held but why it is sometimes held. The present paper addresses thess problems by applying an incomplete information model where a bill is signal. Since one chamber *does* know what the other wants *to some degree*, the bicameral conference is rarely held. But since one chamber does *not* know *exactly* what the other wants, the bicameral conference is sometimes held. Moreover, this model explains when the second chamber amends the first chamber's bill in the first place. Two key factors are uncertainty and importance. When the one chamber's median voter belongs to a different party from the other chamber's, it is not sure of the other chamber's position and is less likely to accept the other chamber's offer with caution. Besides, houses do not hesitate to take an important bill to the conference.

The present paper is organized as follows. The next section introduces a complete information game as a baseline to compare with. In the third section, the article presents its main model of incomplete information game. To begin, perfect Bayesian equilibria and their paths are shown. Next, intuitional interpretation is narrated. The final section concludes. Most of formal argument which the main text discussion is based on is developed in the

Appendix.

COMPLETE INFORMATION MODEL

Setup

Suppose one dimension policy space. Let the ideal point of the first chamber's median voter and that of the second chamber's be denoted by F and S. The game develops as follows.

Nature decides F and S ($F > S$) and reveals these values to both chambers (complete information).

1. The first chamber resolves a bill B_F and sends it to the second chamber.

2. (a) If the second chamber passes it, its version of the bill, B_S, is equal to B_F and is not returned to the first chamber.

   (b) Otherwise, the second chamber amends the bill to $B_S \neq B_F$ and returns it to the first chamber.[1]

3. The game ends in three ways.

   No Amendment: If $B_S = B_F$, the first chamber has nothing to do and $B_S = B_F$ becomes a law, L.

   Acceptance: If the first chamber receives and accepts $B_S \neq B_F$, $L = B_S$.

   Conference: If the first chamber receives but does not accept $B_S \neq B_F$, it calls for the bicameral conference. Any version between the two houses' ideal points can be the final version of the bicameral conference, B_C, with equal chance; namely, B_C follows uniform distribution $\mathcal{U}[S, F]$. For convenience of presentation, it is

[1] If B_S is equal to the status quo Q, it means that the second kills the bill. Unless we consider override option, the game ends here. For the time being, it is supposed that the second chamber never kills a bill.

assumed that conference never fails to reach a conclusion B_C and both chambers always accept it (i.e. prefer it to the status quo). Therefore, $L = B_C$.[2]

In the case of $L = B_F$ or $L = B_S$, utility of each house is the negative value of the distance between its ideal point and the law: $U(F) = -|F - L|$ and $U(S) = -|S - L|$. When the bicameral conference is held, the "conference cost", K_F and K_S (for the first chamber and the second, respectively), are incurred: utility is $U(F) = -|F - L| - K_F$ and $U(S) = -|S - L| - K_S$. The conference cost can be interpreted at least in three ways.

Transaction Cost: Holding the conference and, much more, working out an acceptable report takes effort, time, side payment, etc.

Risk Averse: It is notoriously unpredictable even for senior lawmakers what the final conclusion will look like or whether it comes into existence in the first place. Also, it delays enactment.

Unimportance: If a house has larger stake in the bill, it will not sell it for leisure time but will not dare to go to the conference and make every effort to make the bill best for the house.

Subgame Perfect Nash Equilibrium

Since this is a dynamic game with complete information, equilibrium should be subgame perfect Nash equilibrium. Due to backward induction, the third stage comes first. If the first chamber calls for bicameral conference, its expected utility is

$$\int_F^S \Big(-|F - L| - K_F \Big) \frac{1}{F - S} dL = \frac{F + S}{2} - F - K_F$$

[2] If the conference report is rejected, the status quo continues and $Q = L$. It is assumed that Q is far away enough from both F and S to be preferred to any conference report.

By contrast, if the first chamber accepts B_S, its utility is $B_S - F$. Therefore, if $B_S \geq \frac{F+S}{2} - K_F$, the first chamber accepts the second chamber's bill. Otherwise, the bicameral conference is held. The cutoff point moves from the median between both chambers in the direction of the second chamber by K_F. The first chamber's conference cost does harm the first chamber.

On the second stage, the second chamber prefers the closest bill which the first chamber accepts: $\frac{F+S}{2} - K_F$. This does not depend on what the first chamber sends.

On the first stage, the first chamber may send any bill, because its version does not affect the following stages.

Equilibrium:

1. The first chamber sends any bill B_F.
2. The second chamber resolves $B_S = \frac{F+S}{2} - K_F$.
3. If the first chamber receives $B_S \neq B_F$,
   (a) it accepts $B_S \geq \frac{F+S}{2} - K_F$.
   (b) it does not accept $B_S < \frac{F+S}{2} - K_F$.

Therefore, on the equilibrium path, the bicameral conference is never held because the second chamber, if necessary, amends a bill so that the first chamber accepts it. How far both chambers are from each other does not matter. Moreover, the law is always $\frac{F+S}{2} - K_F$, in favor of the second chamber thanks to the first chamber's conference cost K_F. Finally, what the first chamber sends does not matter.

Two problems are in order. Bicameral conference never occurs in this model, while it does (sometimes) occur in real politics. The second chamber almost always (unless $B_S = B_F$) amends the first chamber's bill according to the model, though it rarely does so in reality. In order to address these, the next section incorporates uncertainty into the model.

INCOMPLETE INFORMATION MODEL

Setup

The model introduced in this section is different from the previous one only in that Nature tells each house its own ideal points only but, at the bicameral conference, both houses realize each other's ideal point. The first chamber, however, has belief, $p(S) = \mathcal{U}[S_E, S_M]$, that the second chamber's ideal point S is uniformly distributed between the lower bound S_E and the upper bound S_M. Similarly, $p(F) = \mathcal{U}[F_M, F_E]$. This uncertainty changes the game dramatically.

Intuition behind this setup is straight-forward and real: a chamber is not sure what the other house really wants. But once they directly negotiate at the conference and must hammer out a take-it-or-leave-it bill, their true preference is revealed to each other. $\Delta_F = F_E - F_M$ and $\Delta_S = S_M - S_E$ represent uncertainty level of the ideal point of the first chamber and the second one, respectively. To be concrete, the most important factor that affects uncertainty level is how different both houses' partisan composition is. In particular, when decisive legislators (the median, agenda setters like the chair, or pivot players such as veto (against veto) on the floor or in the committee) belong to different parties or when only one (typically the lower) house's majority supports the government (divided government in parliamentary system), this uncertainty will be severe.

The value of S represents second chamber's type. When S or B_S is large, they are called "moderate," because they are close to F. By contrast, when they are small, they are "extreme". F means first chamber's type. When F or B_F is small, they are "moderate". When F or B_F is large, they are "extreme". Note that the direction is the opposite between the two chambers. For ease of presentation, suppose that $S_M < F_M$ (This implies that uncertainty of preferences is not so large compared with difference of them).

Perfect Bayesian Equilibria

Since this is a dynamic game with incomplete information, equilibria should be perfect Bayesian equilibria. This paper considers a class of semi-pooling equilibria such that

1. (a) A "Moderate Side" first chamber ($F \leq F^*$) sends "the Compromise Bill" (B_F^*).

   (b) An "Extreme Side" first chamber ($F > F^*$) sends a "Defiant Bill" ($B_F > B_F^0$), expecting no second chamber accepts it.

2. (a) If B_F is the Compromise Bill: an "M-Moderate Side" second chamber ($S \geq S^*$) accepts it. An "M-Extreme Side" second chamber ($S < S^*$) returns a "M-Defiant Amendment" ($B_S < B_S^0$). (Prefix M implies that these second chambers (not necessarily correctly) believe that they face Moderate Side first chambers.)

   (b) If B_F is a Defiant Bill: an "E-Moderate Side" second chamber ($S \geq S^{**}$) returns "the Compromise Amendment" (B_S^{**}). An "E-Extreme Side" second chamber ($S < S^{**}$) returns a "E-Defiant Amendment" $B_S < B_S^{00}$.

   (c) If B_F is an "Off-the-Path Bill" ($B_F^* \neq B_F \leq B_F^0$): (Since this is a off-the-path node, those who are not interested in rigor of this game theoretic model may skip this) if it is more moderate than the "Off-Compromise Amendment" (B_S^{-*}), "Off-Moderate Side" second chambers ($S \geq S^{-*}(B_F)$) accept it, while the other types return an "Off-Defiant Amendment" ($B_S \neq B_S^{-*}$ and $B_S \neq B_S^{-**}$). If a Off-the-Path Bill is more extreme than the Off-Compromise Amendment, Off-Moderate Side second chambers return the Off-Compromise Amendment, while the other types return an Off-Defiant Amendment.

3. If the second chamber returns an amendment ($B_S \neq B_F$) and

   (a) If B_F is the Compromise Bill and B_S is an M-Defiant Amendment: all types of first chambers do not accept it.

(b) If B_F is a Defiant Bill and B_S is the Compromise Amendment: "M-Extreme Side" first chambers ($F \leq F^{**}$) accept it, while "E-Extreme Side" ones ($F > F^{**}$) do not. (Here, Prefixes M implies that these Extreme Side first chambers are the more moderate half among them.)

(c) If B_F is a Defiant Bill and B_S is an E-Defiant Amendment: all types of first chambers do not accept it.

(Since the following three cases are off-the-path nodes, those who are not interested may skip them.)

(d) If B_F is the Compromise Bill and B_S is a M-Off-the-Path Amendment ($B_S^0 < B_S \neq B_S^{**}$): some moderate first chambers ($F \leq F^{-*}(B_F, B_S)$) accept B_S, while the other types do not.

(e) If B_F is Defiant bill and B_S is a E-Off-the-Path Amendment ($B_S^{00} < B_S \neq B_S^{**}$): some moderate first chambers ($F \leq F^{-*}(B_F, B_S)$) accept B_S, while the other types do not.

(f) If B_F is an Off-the-Path Bill: some moderate first chambers ($F \leq F^{-*}(B_F, B_S)$) accept the Off-Compromise Amendment, while the other types do not. No types accept either the Off-Defiant Amendment or any other Off-Off-the-Path Amendment.

Most of technical details are left to the Appendix: the values of these cutoff points, off-the-path belief and proof of the class of equilibria. It will be more helpful for readers to see equilibrium paths, namely, what episodes they would observe if these strategies are played.

No Amendment: A Moderate Side first chamber sends the Compromise Bill. M-Moderate Side Second chambers know that ($p(F|B_F) = \mathcal{U}[F_M, F^*]$) and accept it. The Compromise Bill becomes a law ($L = B_F^*$).

Acceptance: A M-Extreme Side first chamber sends a Defiant Bill. E-Moderate Side Second chambers only know that the first chamber is just Extreme Side ($p(F|B_F) = \mathcal{U}(F^*, F_E]$) and return the Compromise Amendment. The first chamber knows that ($p(S|B_S, B_F) = \mathcal{U}[S^{**}, S_M]$) and accepts the amendment. The Compromise Amendment becomes a law ($L = B_S^{**}$).

Conference: A Moderate Side first chamber sends the Compromise Bill. M-Extreme Side Second chambers know that ($p(F|B_F) = \mathcal{U}[F_M, F^*]$) and return a M-Defiant Amendment. The first chamber knows that ($p(S|B_S, B_F) = \mathcal{U}[S_E, S^{**}]$) and does not accept the amendment. A conference report becomes a law ($L = B_C$).

Conference: An E-Extreme Side first chamber sends a Defiant Bill. E-Moderate Second chambers only know that the first chamber is just an Extreme Side type and return the Compromise Amendment. The first chamber knows that ($p(S|B_S, B_F) = \mathcal{U}[S^{**}, S_M]$) and does not accept the amendment. A conference report becomes a law.

Conference: An Extreme Side first chamber sends a Defiant Bill. E-Extreme Second chambers know that and returns the E-Defiant Amendment. The first chamber knows that ($p(S|B_S, B_F) = \mathcal{U}[S_E, S^{**}]$) and does not accept the amendment. A conference report becomes a law.

It is easy to confirm that on-the-path belief is up to the Bayes Rule and the strategy profiles.

To put more simply, both houses are divided into the moderate side and the extreme side; moderate side types send the compromise offer or accept it; extreme side types send a defiant offer and the other house does not accept it; when both houses are moderate side types, bills are not amended; when the first chamber is neither moderate nor extreme and when the second chamber is moderate side, bills are amended. Otherwise, i.e., when either house is extreme side, the conference is held.

Intuitional Interpretation

The two problems mentioned in the end of the previous section are addressed. This model correctly predicts that the second chamber amends bills not so often; once it does, the first chamber usually, but not always, accepts amendment; or the bicameral conference may occur, but much less often. Difference between the two models arises from introduction of uncertainty.

The more uncertain chamber's ideal point (larger Δ_F or Δ_S), the more likely bills are amended. Moreover, if one chamber is very uncertain of where the other is located (very large Δ_F or Δ_S), the conference is possible. To put it another way, as Δ_F or Δ_S becomes larger, F or S can be more extreme. This sounds straight-forward, though the mechanism is nuanced. It is not just because one house is unfamiliar with the other house, offers bills randomly and sometimes makes an error. Rather, their offer, acceptance and rejection are more systematic. Without buying identity of the opponent chamber, chambers tempt only moderate types of the other chamber to accept its offer. Or, it is ready to pay the conference cost for knowing the exact type of the other chamber if, eventually, it turns out to be extreme.

Another key variable is the conference cost, $K = 4(K_F + K_F)$. As mentioned before, this means transaction cost, risk averse and unimportance of a bill. The more important a bill (small K), the more likely it is to be amended. If a bill is very important (very small K), the conference is possible. Though this relationship may not be obvious, a closer look will persuade readers. When houses do not have a big stake in a bill, it does not find the unimportant bill worth enough to pay the transaction cost or it hates to sell (though imperfect but) certain bills or amendments for risky conference report. By contrast, if a bill is important, chambers can not put up with rough estimate of the other house's ideal point. That is why it amends a bill or takes it to the conference.

Both houses' conference costs equally affect how a law is made (no amendment, acceptance or the conference), though they have the opposite distributional effect on what law is

made. Each house's conference cost does harm that house. That is, when K_F (K_S) increases, B_F^* and B_S^{**} decreases (increases) and the first (second) chamber bears that cost increase. To put it another way, as a house takes a bill seriously (small $K_{F,S}$), it is rewarded. In complete information model, only the first chamber's conference cost is taken advantaged of by the second chamber; in incomplete information model, the opposite is also true. This symmetry seems reasonable.

A bill is signal. It is not just a candidate of a law. It tells the receiver house what type the sender house is. Though the complete information model does not admit, what the first chamber sends does matter for this reason, not just because it is usually accepted as it is.

How far both chambers are from each other does not matter, as the complete information model also argues. This suggests that, when different parties control both house, amendment arises from not their difference but ambivalence of ideal points.

CONCLUSION

In order to explain both absence and presence of the bicameral conference, the role uncertainty plays in the game is critical. Importance of bills is measured against how unsure one house is of the other's intention. Uncertainty and importance encourage both chambers to amend bills and, if it does not suffice, to go to the conference. The incomplete information game model the present paper submits sheds new light on bicameral bargaining.

There remains a lot to improve. To begin, it is definitely mandatory to make the current argument more accessible. Beyond that, the most promising extension is introduction of veto players: president and pivot players (e.g. override, filibuster, and cloture). Bicameral sequence is also an important topic.

As for game theoretic concern, admittedly, this most optimistic belief assumption ($F_A = F_M$) stacks the deck in favor of this strategy profile so that it is an equilibrium. Thus, the

conditions in the present paper are the most generous. If their belief is not so optimistic (namely, if F_A is much more than F_M), some restriction of f^* or f^{**} may be necessary. In addition, electoral consideration can be incorporated into this framework.

APPENDIX

In order to avoid complicated taxonomy, it is assumed that, when acceptance and rejection of a bill bring about the same utility, a player chooses to accept it.

Basic

Some concepts for the first chamber are introduced. Those for the second chambers are their mirror images, unless otherwise noted. Utility of conference for F is

$$\int_F^S \Big(-|F-L| - K_F \Big) \frac{1}{F-S} dL = -\frac{F-S}{2} - K_F$$

This is the negative value of distance between two chambers minus F's conference cost. When the first chamber has belief $p(S) = [\underline{S}, \bar{S}]$, define "average type" as

$$S_A(p(S)) \equiv \frac{\underline{S} + \bar{S}}{2}.$$

Utility of conference for F with $p(S)$ is

$$\int_{\underline{S}}^{\bar{S}} \Big(-\frac{F-S}{2} - K_F \Big) \frac{1}{\bar{S} - \underline{S}} dS = -\frac{F - S_A}{2} - K_F$$

That is, S's ideal point is replaced by its average type. Define "Conference Equivalent Bill" $B^C(F|p(S))$ as the bill whose utility is the same as expected utility of conference and which is in the direction of the other chamber.

$$\begin{aligned} -(F - B^C(F|p(S))) &= -\frac{F - S_A}{2} - K_F \\ \therefore B^C(F|p(S)) &= \frac{F + S_A}{2} - K_F \\ B^C(S|p(F)) &= \frac{F_A + S}{2} + K_S \end{aligned}$$

Note that the the signs of conference cost are different between chambers.

It is assumed that $p(S)$ is not dependent on F (every type has the same belief).

Proposition 1: Consider the case of the third stage where the first chamber F receives a bill B_S from the second chamber with the belief $p(S)$. Suppose that, given $p(S)$, F^0 accepts B^0 on the third stage in the equilibrium. Then, a more moderate type ($F^0 \geq F^{00} \geq 2B^0 - F^0$) also accepts B^0 in the equilibrium.

Proof: Since B^0 is closer to F^0 than its conference equivalent bill, it follows

$$\begin{aligned} B^0 &\geq B^C(F^0|p(S)) \\ &= \frac{F^0 + S_A}{2} - K_F \\ &\geq \frac{F^{00} + S_A}{2} - K_F \\ &= B^C(F^{00}|p(S)). \end{aligned}$$

This establishes the proposition.

Lemma 1: If F^0 does not accept B^0 on the third stage in the equilibrium, a more extreme type ($F^{00} \geq F^0$) does not accept B^0 in the equilibrium, either.

Proof: Contraposition of the proposition.

Proposition 2: $S_e \leq S \leq S_m$ believes that $F_m \leq F \leq F_e F$. S has two options. If it chooses B^+ and $F \leq F^+$, B^+ becomes a law. If $F > F^+$, a conference report becomes a law. Similarly, if it chooses B^- and $F \leq F^-$, B^- becomes a law. If $F > F^-$, a conference report becomes a law. Let

$$\Delta U(S) = \frac{1}{4}\Big((f^+(f^+ + \Delta b^+) - f^-(f^- + \Delta b^-)\Big)$$

where $f^+, f^-, \Delta b^+$ and Δb^- are defined so that

$$f^+ = F^+ - F_m$$

$$f^- = F^- - F_m$$

$$\Delta b^+ = 4\Big((\frac{F_m + S}{2} + K_S) - B^+\Big)$$

$$\Delta b^- = 4\Big((\frac{F_m + S}{2} + K_S) - B^-\Big)$$

Then, the condition for all types of $S \leq S_m$ to prefer B^+ to B^- is $\Delta U(S_m) \geq 0$.

Proof: The expected utility for S to choose B^+ is

$$U^+(S) = \int_{F_m}^{F^+} -|B^+ - S|\frac{1}{F_e - F_m}dF + \int_{F^+}^{F_e}(-\Big|\frac{F+S}{2} - S\Big| - K_S)\frac{1}{F_e - F_m}dF.$$

The expected utility for S to choose B^- is s

$$U^-(S) = \int_{F_m}^{F^-} -|B^- - S|\frac{1}{F_e - F_m}dF + \int_{F^-}^{F_e}(-\Big|\frac{F+S}{2} - S\Big| - K_S)\frac{1}{F_e - F_m}dF.$$

By a bit algebra, it is shown that $U^+(S) - U^-(S) = \Delta U(S)$. Then, if $\Delta U(S_m) \geq 0$, S_m prefers B^+ to B^-. Therefore, according to Proposition 1, all types of $S \geq S_m$ also prefers B^+ to B^-. Q.E.D.

Lemma 2: Change the side of chambers by replacing comparable parameters in Proposition2 with negative sign: $F = -S, F_m = -S_m, F^+ = -S^+, F_e = -S_e, B^+ = -B_+, S_m = -F_m, S_e = -F_e, F^- = -S^-, B^- = -B_-$ and inequality is inversed. Let

$$\Delta U(F) = \frac{1}{4}\Big((s^+(s^+ + \Delta b^+) - s^-(s^- + \Delta b^-)\Big)$$

where

$$s^+ = S_m - S^+$$

$$s^- = S_m - S^-$$

$$\Delta b^+ = 4\Big((\frac{S_m + F}{2} - K_F) - B^+\Big)$$

$$\Delta b^- = 4\Big((\frac{S_m + F}{2} - K_F) - B^-\Big)$$

Then, the condition for all types of $F \geq F_m$ to prefer B^+ to B^- is $\Delta U(F_m) \geq 0$.

Proof: The mirror image of the proof of Proposition 2.

Indifferent Types and the Compromise Offers

<u>Bill Transfer from the Second Chamber to the First Chamber.</u> Given the first chamber's equilibrium strategy, after receiving a Defiant Bill, the utility for the type S of second chamber to send the Compromise Amendment is

$$\int_{F^*}^{F^{**}} -|B_S^{**} - S|\frac{1}{F_E - F^*}dF + \int_{F^{**}}^{F_E}\Big(-\frac{|F - S|}{2} - K_S\Big)\frac{1}{F_E - F^*}dF$$

The S's utility to send a Defiant Amendment and go to the conference is

$$\int_{F^*}^{F_E}\Big(-\frac{|F - S|}{2} - K_S\Big)\frac{1}{F_E - F^*}dF$$

For the "E-Indifferent Type" of second chamber ($S = S^{**}$), both utility values should be the same. Since it is assumed that $B_S^{**} > S^{**}$ and $F \geq F_M > S_M \geq S^{**}$,

$$\begin{aligned}
&\int_{F^*}^{F^{**}} \Big(-(B_S^{**} - S^{**}) + (\frac{F - S^{**}}{2} + K_S)\Big) \frac{1}{F_E - F^*} dF \\
&= \frac{F^{**} - F^*}{F_E - F^*} \frac{1}{F^{**} - F^*} \Big[\frac{F^2}{4} + (-B_S^{**} + \frac{S^{**}}{2} + K_S)F \Big]_{F^*}^{F^{**}} \\
&= 0 \\
\therefore B_S^{**} &= \frac{\frac{F^{**}+F^*}{2} + S^{**}}{2} + K_S
\end{aligned}$$

Thus, the Compromise Amendment is the conference equivalent bill of the Indifferent Type second chamber who believes that the first chamber is an Extreme Side type. Unlike the complete information model, the second chamber's conference cost (K_S) matters: the larger K_S, the farther the Compromise Amendment is from second chambers. It does harm the second chamber. Obviously, E-Moderate Side second chambers prefer the Compromise Amendment to Defiant Amendments, while E-Extreme Side second chambers prefer the opposite. Thus, S's strategy is incentive compatible.

Similarly, given the second chamber's equilibrium strategy, after receiving the Compromise Amendment, the "E-Indifferent Type" of first chamber (F^{**}) have the same utility of accepting the Compromise Amendment as that of rejecting it. It follows

$$\begin{aligned}
&\int_{S^{**}}^{S_M} \Big(-(F^{**} - B_S^{**}) + (\frac{F^{**} - S}{2} + K_F)\Big) \frac{1}{S_M - S_E} dF \\
&= 0 \\
\therefore B_S^{**} &= \frac{\frac{S^{**}+S_M}{2} + F^{**}}{2} - K_F
\end{aligned}$$

Its interpretation is the mirror image of the previous paragraph.

From the precedent two paragraphs,

$$
\begin{aligned}
B_S^{**} &= \frac{\frac{F^{**}+F^*}{2} + S^{**}}{2} + K_S \\
&= \frac{\frac{S_M+S^{**}}{2} + F^{**}}{2} - K_F \\
\therefore f^{**} + s^{**} &= K
\end{aligned}
$$

where $K \equiv 4(K_F + K_S), f^{**} \equiv F^{**} - F^*, s^{**} \equiv S_M - S^{**},$. This implies that uncertainty of both chambers is equal to the total conference cost.

<u>Bill Transfer from the First Chamber to the Second Chamber.</u> The "M-Indifferent Type" second chamber (S^*) has the same utility of accepting the Compromise Bill as that of returning M-Defiant Amendment and going to the conference. It follows

$$
\begin{aligned}
&\int_{F_M}^{F^*} \Big(-(B_F^* - S^*) + \Big(\frac{F - S^*}{2} + K_S\Big)\Big) \frac{1}{F_* - F_M} dF \\
&= 0 \\
\therefore B_F^* &= \frac{\frac{F^*+F_M}{2} + S^*}{2} + K_S
\end{aligned}
$$

Obviously, M-Moderate Side second chambers prefer the Compromise Bill to M-Defiant Amendments, while M-Extreme Side second chambers prefer the opposite. Thus, S's strategy is incentive compatible.

The "M-Indifferent Type" first chamber ($F = F^*$) has the same utility of sending the Compromise Bill (which may be accepted or lead to the Conference) as that of sending Defiant Bill (which may result in the Compromise Amendment or the Conference). Applying Lemma 2 where $F = F^*, S_m = S_M, S^+ = S^*, s^+ = s^*, F_m = F_M, F_e = F_E, B^+ = B_F^*, s^- =$

$s^{**}, B^- = B_F > B_F^{-0}$, one obtains

$$\begin{aligned}\Delta U(f^*) &= \frac{1}{4}(s^*(s^* + (f^* - K)) - s^{**}(s^{**} - (2f^{**} + s^{**} \\ &= 0 \\ \therefore f^* &= K - s^* - \frac{2s^{**}}{s^*}(f^{**} - s^{**})\end{aligned}$$

If there is no M-Moderate second chambers ($s^{**} = 0$), this reduces to $f^* + s^* = K$, which is equivalent to $f^{**} + s^{**} = K$. The more advantageous the Compromise Amendment (larger $f^{**} - s^{**}$) or the more M-Moderate second chambers, the fewer Moderate Side first chambers. In words, the better prospect of the Compromise Amendment first chambers have, they do not have to cling to the Compromise Bill. Since $\Delta U(f < f^*) > 0$, Moderate Side first chambers prefer the Compromise Bill to Defiant Bills. Since $\Delta U(f > f^*) < 0$, Extreme Side ones prefer the opposite. Thus, F's strategy is incentive compatible.

Defiant Offer

Defiant Bills should be too extreme for any second chamber type to accept. Otherwise, some second chambers have incentive to deviate from the equilibrium strategy. Given the first chamber's equilibrium strategy, after receiving a Defiant Bill, the utility for the type S_M of second chamber to return the Compromise Amendment is

$$\int_{F^*}^{F^{**}} -|B_S^{**} - S|\frac{1}{F_E - F^*}dF + \int_{F^{**}}^{F_E} \Big(- \frac{|F - S|}{2} - K_S \Big)\frac{1}{F_E - F^*}dF$$

The S's utility to accept the most moderate Defiant Amendment B_F^0 is $-|B_F^0 - S|$. The easiest to tempt to defect is the most moderate type of second chambers, S_M. Suppose that $S = S_M$ is indifferent between accepting B_F^0 and returning the Compromise Amendment. It

follows

$$B_F^0 = \frac{F^{**} - F^*}{F_E - F^*} B_S^{**} + \frac{F_E - F^{**}}{F_E - F^*} \Big(\frac{\frac{F_E + F^{**}}{2} + S_M}{2} + K_S \Big)$$

The last term is the Conference Equivalent Bill of S_M who believes that the first chamber is E-Extreme Side type. Thus, B_F^0 is the proportional division between that Conference Equivalent Bill and the Compromise Amendment. Reasonably, Defiant Bills are more extreme than the Compromise Bill: $B_F \geq B_F^0 \geq B_S^{**} > B_F^*$. According to Proposition 1, all types of second chambers ($S \leq S_M$) do not accept $B_F > B_F^0 > B_F^*$, which is called Defiant bill.

Similarly, when an M-Extreme second chamber receives the Compromise Bill, it should return an M-Defiant Amendment ($B_S < B_S^0$) which is too extreme for any Moderate Side first chamber to accept. The most moderate type first chamber (F_M) should be indifferent between accepting the most moderate M-Defiant Amendment (B_S^0) and rejecting it, believing that the second chamber is M-Extreme Side. This means that B_S^0 is the conference equivalent bill.

$$\begin{aligned} B_S^0 &= B^C(F_M | p(S) = \mathcal{U}[S_E, S^*]) \\ &= \frac{\frac{S_E + S^*}{2} + F_M}{2} - K_F \\ &\leq \frac{\frac{S_M + S^*}{2} + F^*}{2} - K_F \\ &= B_F^* \end{aligned}$$

This is more extreme than the Compromise Bill. Then, all types of $F \geq F_M$ do not accept M-Defiant Amendment $B_S < B_S^0$.

When an E-Extreme second chamber receives an E-Defiant Bill, an amended bill should be too extreme for any extreme first chamber type to accept. The most moderate type first

chamber to send a Defiant Bill in the equilibrium is the Indifferent Type (F^*). Following the previous paragraph, the most moderate E-Defiant Bill is the conference equivalent bill of the Indifferent Type who believes that the second chamber is E-Extreme Side.

$$\begin{aligned} B_S^{00} &= B^C(F^*|p(S) = \mathcal{U}[S_E, S^{**}]) \\ &= \frac{\frac{S_E+S^{**}}{2} + F^*}{2} - K_F \\ &\leq \frac{\frac{S_M+S^{**}}{2} + F^*}{2} - K_F \\ &= B_S^{**} \end{aligned}$$

This is more extreme than the Compromise Amendment. Then, all extreme types of $F \geq F_M$ do not accept E-Defiant Amendment $B_S < B_S^{00}$.

Off-the-Path Behavior

This subsection makes clear the conditions that chambers do not have incentive to choose an Off-the-Path Bill or Amendment.

<u>Off-the-Path Amendment.</u> Let $F^{-*}(B_F, B_S) = 2(B_S + K_F) - S_A(B_F, B_S)$ and $s_A = S_M - S_A$. Like the above consideration of Off-the-Path Bill, it is not necessary to consider those Off-the-Path Amendments which all (possible) first chambers types accept or reject. It is the best response for $F \leq F^{-*}((B_F, B_S)$ to accept an Off-the-Path Bill B_S and for $F > F^{-*}(B_F, B_S)$ to reject it. Is it also the best response for second chambers not to send an Off-the-Path Amendment? According to Proposition 1, yes it is, if even for the easiest to defect, namely, the most moderate type of second chambers, does not have incentive to defect.

If B_F is the Compromise Bill: According to the first chamber's strategy and the Bayes Rule, the second chambers believe that the first chamber is a Moderate Side type.

Consider whether M-Moderate Side second chambers accept the Compromise Bill or return an M-Off-the-Path Amendment. Appling Proposition 2 where $F_m = F_M, f^+ = f^*, F_e =$

$F^*, S_m = S_M, S_e = S^*, B^+ = B_F^*, f^- = F^{-*} - F_M = f^{-*}, B^- = B^{-*}$, one obtains

$$\Delta U(f^{-*}) = \frac{1}{4}(-f^{-*2} - (K + 2s_A)f^{-*} + 2f^*(f^* + s^*))$$

Since $\Delta U(f^{-*})$ is concave for f^{-*}, $\Delta U(f^{-*}) \geq 0$ for $0 \leq f^{-*} \leq F^* - F_M$ if $\Delta U(0) \geq 0$ and $\Delta U(F^* - F_M) \geq 0$. $\Delta U(F_M - F_M = 0) = 2f^*(f^* + s^*) \geq 0$ is always true. In order that $\Delta U(F^* - F_M) = f^*(f^* - (K - 2(s^* - s_A))) \geq 0$, it is necessary $s^* \geq K/2 + s_A$. This is the condition for M-Moderate Side second chambers not to defect.

Next, do M-Extreme Side second chambers return Defiant Amendment or Off-the-Path one? Appling Proposition 2 where $F_m = F^+ = F_M, f^+ = 0, F_e = F^*, S_m = S^*, S_e = S_E, B^+ < B_S^0, f^- = f^{-*}, B^- = B^{-*} = \frac{S_M + F^{-*}}{2} - K_F$, one obtains

$$\Delta U = \frac{1}{4}f^{-*}(f^{-*} - (K - 2(s^* - s_A)))$$

In order that $\Delta U(f^*) \geq 0$ for $F_M \leq F^{-*} \leq F_E$, it is necessary $s^* \geq K/2 + s_A$.

If B_F is a Defiant bill, the second chambers believe that the first chamber is an Extreme Side type. Which do E-Moderate Side second chambers return, the Compromise Amendment or an E-Off-the-Path Amendment? Appling Proposition 2 where $F_m = F^*, f^+ = f^{**}, F_e = F_E, S_m = S_M, S_e = S^{**}, B^+ = B_S^{**}, f^- = F^{-**} - F^* = f^{-**}, B^- = B^{-**}$, one obtains

$$\Delta U(f^{-**}) = \frac{1}{4}(-f^{-**2} - (K + 2s_A)f^{-**} + 2f^{**}(f^{**} + s^{**}))$$

One obtains this expression by replacing comparable parameters of the expression for M-Moderate Side second chambers. Thus, the conditions not to defect are $\Delta U(f^{**}) > 0$ and $\Delta U(f_E \equiv F_E - f^*) > 0$. The former is met when $s^{**} \geq K/2 + s_A$. Due to $f^{**} + s^{**} + K$, the latter requires

$$f^{**} > \frac{f_E^2 + (K + 2s_A)f_E}{2K}$$

The case of E-Extreme Side second chambers also is comparable to that of M-Extreme Side ones. Appling Proposition 2 where $F_m = F_*, f^+ = 0, F_e = F_E, B^+ < B_S^{00}, S_m = S^{**}, S_e = S_E, f^- = f^{-**}, B^- = B^{-**}$ or replacing comparable parameters of the expression for M-Extreme Side second chambers,

$$\Delta U = \frac{1}{4} f^{-**}(f^{-**} - (K - 2(s^{**} - s_A))) \geq 0$$

must be satisfied for $F_M \leq F^{-**} \leq F_E$. It is necessary $s^{**} \geq K/2 + s_A$.

<u>Off-the-Path Bill.</u> What do second chambers do if F sends an Off-the-Path Bill B_F? It is assumed that S (mostly too optimistically) believes $F = F_M$. If B_F is more moderate for S than the "Off-Compromise Amendment" $B_S^{-*} = B^C(F_M|S^{-**} \leq S \leq S_M) = B^C(S^{-**}|F_M)$ (where $S^{-**} = S_M - K$), $S \geq S^{-0}(B_F < B_S^{-*}) = 2(B_F - K_S) - F_M$ accepts it and $S < S^{-0}$ returns an Off-Defiant Amendment ($B_S < B_S^{-0}(B_F < B_S^{-*}) = B^C(F_M|S_E \leq S \leq S^{-0})$, where $S^{-0} < S^{-**}$. If B_F is more extreme for S than the Off-Compromise Amendment, $S^{-**} \leq S \leq S_M$ returns the Off-Compromise Amendment and $S < S^{-**}$ returns an "Off-Defiant Amendment" ($B_S < B_S^{-0}(B_F \geq B_S^{-*}) = B^C(F_M|S_E \leq S \leq S^{-**})$.

According to this S's strategy, the Bayes Rule and the off-the-path belief given below, F's best response on the third stage is as follows. Note that $B^C(F|p(S))$ increases in F and S_A and $B^C(S|p(F))$ increases in S and F_A. Also note that, on the third stage, F accepts B_S if and only if $B_S \geq B^C(F|p(S))$.

1. After F receives the Off-Compromise Amendment, F correctly believes $S^{-**} \leq S \leq S_M$. F_M accepts it because it is its Conference Equivalent Bill. The other types of F does not accept it because $B^C(F > F_M|S^{-**} \leq S \leq S_M) > B^C(F_M|S^{-**} \leq S \leq S_M) = B_S^{-*}$.

2. When F receives an Off-Defiant Amendment (B_S) and $B_F \geq B_S^{-*}$, F correctly believes $S < S^{-**}$ and does not accept it because $B_S < B^C(F_M|S_E \leq S \leq S^{-**}) < B^C(F >$

$F_M|S_E \leq S \leq S^{-**}$). When $B_F < B_S^{-*}$, replace S^{-**} by S^{-0}.

3. After F receives an "Off-Off Amendment" ($B_S^0 \leq B_S \neq B_S^{-*}$), it is assumed that F has the off-the-path belief $S = S_M$. F accepts $B_S \geq B^C(F|S_M)$ but not other Off-Defiant Amendments.

Is the S's strategy also the best response to these F's strategy and off-the-path belief? Consider the case where $B_F \geq B_S^{-*}$. Note that, if S believes $F = F_M$, S prefers B to the conference if and only if $B \leq B^C(S|F_M)$.

1. If $S^{-**} \leq S \leq S_M$ returned such an Off-the-Path Amendment as $B_S \geq B^C(F_M|S = S_M)$, S would believe that F_M would accept it. But this amendment is more extreme than the Off-Compromise Amendment $B^C(F_M|S^{-**} \leq S \leq S_M)$. If these S's returned one of the other group of Off-the-Path Amendments ($B_S < B^C(F_M|S = S_M)$) or an Off-Defiant Amendment, S would believe that F_M would not accept it and the conference is held. But their Conference Equivalent Bill ($B^C(S^{-**} \leq S \leq S_M < S_M|F_M)$) is more extreme than the Off-Compromise Amendment ($B^C(S^{-**}|F_M)$). Therefore, these S's do not defect from the Off-Compromise Amendment.

2. If $S_E \leq S < S^{-**}$ returned the Off-Compromise Amendment ($B^C(S^{-**}|F_M)$), S would believe that F_M would accept it. But this amendment is more extreme (larger) than their Conference Equivalent Bill ($B^C(S_E \leq S < S^{-**}|F_M)$). If these S's returned an Off-the-Path Amendment and it is accepted, that is even worse than the Off-Compromise Amendment, as argued in the previous paragraph. Even if rejected, it is no better than Off-Defiant Amendments. Therefore, these S's do not defect from Off-Defiant Amendments.

When $B_F < B_S^{-*}$, read S^{-**} as S^{-0}. Therefore, given F's off-the-path belief and (correspondingly rational) strategy, S does not have incentive to defect from the equilibrium strategy.

Is it also the best response for first chambers not to send an Off-the-Path Bill on the first stage? Consider the case of Moderate Side first chambers. Applying Lemma 2 where $F = F_M, S_m = S_M, S^+ = S^*, s^+ = s^*, F_m = F_M, F_e = F^*, B^+ = B_F^*, s^- = s^{-0} \geq s^{**} = K, B^- = B_S^{-*}$ (Off-the-Path Amendment is the case of $s^{-0} = s^{**}$), one obtains

$$\Delta U(f^*) = \frac{1}{4}(s^*(s^* + (-f^* - K)) - s^{-0}(s^{-0} + (K - 2s^{-0}))$$

$$= 0$$

$$\therefore s^* = \frac{K + \sqrt{f^{**} + 3s^{**2}}}{2}$$

If this condition is met, according to proposition 1, all Moderate Side first chamber types never defect from the Compromise Bill.

How about Extreme Side first chambers? Since the Indifferent Type first chamber belongs to Moderate Side and is indifferent between the Compromise Bill and Defiant Bill, the previous paragraph implies that this type does not defect. Moreover, this type belongs to Extreme Side as well. According to proposition 1, all Moderate Side first chamber types never defect from the Compromise Bill.