

Estimating Incumbency Advantage
and Campaign Spending Effect
without the Simultaneity Bias:
Strategic Incumbents, Challengers, and Contributors *

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Abstract

In estimating incumbency advantage and campaign spending effect, simultaneity problem is composed of stochastic dependence and parametric dependence. Scholars have tried to solve the former, while the present paper intends to tackle the latter. Its core idea is to estimate parameters by maximizing likelihood of all endogenous variables (vote, both parties' candidate qualities and campaign spending) simultaneously.

In order to do it, I take advantage of theories of electoral politics rigorously, model each

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endogenous variables by the others (or their expectation), derive Bayesian Nash equilibria, and plug them into my estimator. I show superiority of my model compared to the conventional estimators by Monte Carlo simulation. Empirical application of this model to the recent U.S. House election data demonstrates that incumbency advantage is smaller than previously shown and that entry of incumbent and strong challenger is motivated by electoral prospect.

1 Introduction

Ordinary Americans take it for granted that incumbents have advantage in the U.S. House election and large campaign spending helps them, though, surprisingly, political scientists have trouble in measuring their size because of “simultaneity bias”. The logic is as follows. On one hand, when incumbent legislators foresee its defeat, they do not run for reelection. They are strategic. Only incumbents who expect they will win run. As a result, incumbency advantage is overestimated. On the other hand, those incumbents who have poorer electoral prospect need to and do raise and spend more campaign fund but still end up with not so many votes. Thus, it *seems* as if the more campaign contribution lead to the less votes. In this sense, incumbent’s campaign spending effect is underestimated. For both aspects, causal direction between vote and incumbency or money is not only from the latter to the former but also in the opposite way. That is why this is called simultaneity bias.

Simultaneity problem is composed of stochastic dependence and parametric dependence. Scholars have tried to solve the former, while the present paper intends to tackle the latter. Its core idea is to estimate parameters by maximizing likelihood of all endogenous variables simultaneously. In order to do it, I take advantage of theories of electoral politics rigorously, construct a game theoretic model, and plug it into my estimator.

This paper is organized as follows. The first section explains the simultaneous bias problem and previous solutions. Next, I derive Bayesian Nash equilibria of the candidate selection game and illustrate my statistical model. Third, Monte Carlo simulation is demonstrated. The following section will analyze U.S. House election data. Finally, I conclude.

2 Simultaneity Bias

At first, I introduce my notation of variables. Players are a defender party D and a challenger party C . Party’s choice of candidate is denoted by binary variable $Q_{D, C}$; when it fields a

high quality candidate, $Q_{D,C} = 1$; when it fields a low quality candidate, $Q_{D,C} = 0$. For defender, a high quality candidate is equal to incumbent legislator. This notation makes presentation simple. Even though the word “incumbent” is usually used for party and candidate, this paper uses it only for candidate but not party and distinguishes defender party and incumbent candidate for clarification of argument. For candidate quality of the challenger party, the electoral studies almost agree to use prior experience of elective office as its proxy (Bianco, 1984; Cox and Katz, 2002; Jacobson and Kernell, 1983). In addition, both parties spend campaign money $M_{D,C}$ (logged) to boost each vote. x is a vector of all the other covariates such as national tide (dummy of democrat). At last, the defender and the challenger receive two-party vote share V_D and V_C . A large letter refers to a variable (e.g. $Q_{D,C}$), while a small letter refers to its observed value (e.g. $q_{D,C}$).

2.1 Problem

2.1.1 Incumbency Advantage

Today, the canonical estimator of incumbency advantage is Gelman and King (1990)’s. They propose to regress defender’s vote on incumbent candidate dummy, defender party indicator P , and lagged vote $V_{D,t-1}$:

$$\begin{aligned} V_D &= f_V + \epsilon_V \\ f_V &= \beta_0 + \beta_{QD}Q_D + \beta_P P + \beta_V V_{D,t-1} \\ \epsilon_V &\sim N(0, \sigma_V^2) \end{aligned}$$

where P is 1 if the defender is Democrat and -1 if it is Republican. Then, the effect (β_{QD}) of incumbency status (Q_D) of defender party’s candidate is their estimate of incumbency advantage and it is estimated by least square.

This estimator, however, implicitly assumes stochastic independence and parametric de-

pendence between V_D and Q_D (King, 1989, 190-91). The data generation process is similar to that of Heckman (1974)'s sample selection model. Before explaining that, I define a latent vote variable which an incumbent expects to receive if it runs:

$$\tilde{V}_D(Q_D = 1) = f_V(Q_D = 1, \beta) + \tilde{\epsilon}_V$$

Suppose that an incumbent retires strategically (Cox and Katz, 2002; Jacobson and Kernell, 1983). When it believes its vote will be over the defender's threshold vote \tilde{v}_D^* (e.g. $\tilde{V}_D(Q_D = 1) > \tilde{v}_D^* = 0.5$), it runs ($q_D = 1$). Otherwise, it retires ($q_D = 0$).

Consider the joint distribution of V_D and $\tilde{V}_D(Q_D = 1)$ conditioned on x (including P and $V_{D,t-1}$) and β (including β_{Q_D}).

$$\begin{aligned} (V_D, \tilde{V}_D|x, \beta) &\sim p(v_D, \tilde{v}_D|x, \beta) \\ &= p(v_D|\tilde{v}_D = f_V(Q_D = 1, x, \beta) + \tilde{\epsilon}_V) \times p(\tilde{v}_D|x, \beta) \end{aligned}$$

If we assume ϵ_V is independent of $\tilde{\epsilon}_V$ (statistical independence),

$$p(v_D, \tilde{v}_D|x, \beta) = p(v_D|f_V(Q_D = 1, x, \beta)) \times p(\tilde{v}_D|x, \beta)$$

Unfortunately, we do not observe \tilde{v}_D but Q_D . From the above, we can obtain the joint

distribution of V_D and Q_D ,

$$\begin{aligned}
(V_D, Q_D | x, \beta) &\sim p(v_D, q_D | x, \beta) \\
&= p_1(v_D)^{q_D} p_0(v_D)^{1-q_D} \\
p_1(v_D) &= \int_{\tilde{v}_D^*}^{\infty} p(v_D, \tilde{v}_D | x, \beta) d\tilde{v}_D \\
&= p(v_D | f_V(Q_D = 1, x, \beta)) \int_{\tilde{v}_D^*}^{\infty} p(\tilde{v}_D | x, \beta) d\tilde{v}_D \\
p_0(v_D) &= \int_{-\infty}^{\tilde{v}_D^*} p(v_D, \tilde{v}_D | x, \beta) d\tilde{v}_D \\
&= p(v_D | f_V(Q_D = 0, x, \beta)) \int_{-\infty}^{\tilde{v}_D^*} p(\tilde{v}_D | x, \beta) d\tilde{v}_D
\end{aligned}$$

Gelman and King (1990) also implicitly assume that $p(\tilde{v}_D)$ is independent of β (parametric independence). Then,

$$\begin{aligned}
p(v_D, \tilde{v}_D | x, \beta) &= p(v_D | f_V(Q_D = 1, x, \beta)) \times p(\tilde{v}_D | x) \\
p_1(v_D) &= p(v_D | f_V(Q_D = 1, x, \beta)) \int_{\tilde{v}_D^*}^{\infty} p(\tilde{v}_D | x) d\tilde{v}_D \\
p_0(v_D) &= p(v_D | f_V(Q_D = 0, x, \beta)) \int_{-\infty}^{\tilde{v}_D^*} p(\tilde{v}_D | x) d\tilde{v}_D
\end{aligned}$$

Thus, β can be estimated by maximizing likelihood of v_D or minimizing $(v_D - f_V)^2$.

This way, this estimator relies on assumption of stochastic independence and that of parametric independence between V_D and Q_D . But they are problematic. First, unless there is no model specification error, errors ϵ_V and $\tilde{\epsilon}_V$ are correlated (Heckman, 1979). Second, as incumbency advantage β_{Q_D} increases, expected incumbent's vote $\tilde{v}_D(Q_D = 1) = f_V(Q_D = 1, \beta) + \tilde{\epsilon}_V$ also increases and, therefore, an incumbent ($Q_D = 1$) is more likely to run for reelection ($\tilde{v}_D(Q_D = 1) > \tilde{v}_D^*$). Therefore, maximizing likelihood of v_D only does not take into consideration incumbency effect on incumbent emergence. Thus, stochastic dependence and parametric dependence are problems.

2.1.2 Challenger Candidate's Quality

The above argument also holds for high quality challenger's effect on vote (β_{QC}). The larger $-\beta_{QC}$ or $-\tilde{\epsilon}_V$ is, expected challenger's vote $\tilde{v}_C(Q_C = 1)$ increases (Green and Krasno, 1988; Jacobson and Kernell, 1983) and, therefore, a strong candidate of the challenger party ($Q_C = 1$) is more likely to run ($-\tilde{v}_D(Q_C = 1) > \tilde{v}_D^{**}$, where \tilde{v}_D^{**} is the challenger's threshold vote). The challenger is also a strategic player (Bond, Covington and Fleisher, 1985; Jacobson and Kernell, 1983).

2.1.3 Campaign Spending Effect

Defender's campaign spending effect β_{MD} is crucial, though its measurement is controversial. Jacobson (1989, 1990) reports that challenger's campaign spending diminishes defender's vote V_D ($\beta_{MC} < 0$), while defender's has no effect ($\beta_{MD} = 0$). Since then, a lot of scholars have tried to find that defender's war chest also matters (Erikson and Palfrey, 1998, 2000; Goidel and Gross, 1994; Green and Krasno, 1988; Kenny and McBurnet, 1994; Levitt, 1994).

V_D and $M_{D,C}$ affect each other simultaneously, though their relationship is not as straight-forward as that between V_D and $Q_{D,C}$. Suppose that the more money candidates spend, the more vote they receive. Unlike the case of candidate quality, an effect of expected vote (\tilde{v}_D) on campaign spending depends on not its level but its closeness or competitiveness. On one hand, when they foresee their vote is nearly 50 %, they definitely need to expend more. On the other hand, when they are almost sure to win or lose, marginal increase of votes by additional spending is not worth its cost for strategic contributors and candidates (Jacobson and Kernell, 1983). Erikson and Palfrey (2000, 599) formally show that "equilibrium candidate spending should be proportional to the normal density of the expected incumbent margin of victory." Besides, simultaneity also exists between $Q_{D,C}$ and $M_{D,C}$.

2.2 Previous Solutions to Stochastic Dependence

So far, scholars have tried to solve stochastic dependence but it is difficult. As I mentioned above, the relation between V_D and $Q_{D, C}$ is typical sample selection situation. Heckman (1974)'s sample selection model is, however, unavailable due to exclusion restriction because the same covariates should affect both (Sartori, 2003).

The most common method is to employ instrumental variable (Erikson and Palfrey, 1998; Green and Krasno, 1988; Kenny and McBurnet, 1994). To find appropriate instrumental variable itself is, however, problematic task.

Another way is to utilize natural experiment. Levitt (1994) and Levitt and Wolfram (1997) examine elections where the same two candidates face one another on more than one occasion to control all time invariant district specific features and candidate specific ones, observed or unobserved or unobservable. Cox and Katz (2002) pay attention to a non incumbent's vote in such a district where the incumbent fails to run involuntarily (namely, not for electoral reason) because it is a good estimate of the vote the incumbent would receive if it run as non incumbent.

2.3 A Solution to Parametric Dependence

Unlike previous endeavors, this paper aims to solve parametric dependence (my model can not fail to assume stochastic independence, though). By making much of electoral studies and constructing a game theoretic model, I will specify how covariates x and β affects not only V_D but also other endogenous variables ($Q_{D, C}$ and $M_{D, C}$) parametrically. When I model each endogenous variable, I take into account expectation of the other endogenous variables (e.g. $\tilde{v}(q_D, q_C, x), \tilde{q}_{D, C}(x)$). My model estimates parameters by maximizing the likelihood

of the five endogenous variables simultaneously, which is derived by their joint distribution.

$$\begin{aligned}
L(V, Q_D, Q_C, M_D, M_C|x) &\propto p(v, q_D, q_C, m_D, m_C|x) \\
&= \mathcal{N}(v|q_D, q_C, m_D, m_C, x) \\
&\quad \times \mathcal{N}(m_D|q_D, q_C, x, \tilde{v}(q_D, q_C, x)) \times \mathcal{N}(m_C|q_D, q_C, x, \tilde{v}(q_D, q_C, x)) \\
&\quad \times \mathcal{B}(q_D|x, \tilde{q}_D(x), \tilde{q}_C(x), \tilde{v}(\tilde{q}_D(x), \tilde{q}_C(x), x)) \\
&\quad \times \mathcal{B}(q_C|x, \tilde{q}_D(x), \tilde{q}_C(x), \tilde{v}(\tilde{q}_D(x), \tilde{q}_C(x), x)),
\end{aligned}$$

where \mathcal{N} and \mathcal{B} represent normal and Bernoulli distributions, respectively.

Goidel and Gross (1994) also estimate system of four equations (V_D, Q_C, M_D, C) simultaneously by three-stage least square. A problem of their model is failure to take into consideration expectation of endogenous variables. For example, they do not include expected vote into the equation of candidate quality. Since their equations share some covariates but not parameters, their model implicitly assume parametric independence.

3 Model

3.1 Vote Share Log Odds: V

3.1.1 Log Odds

Though scholars use defender's vote share V_D or its margin $V_D - V_C$, this paper prefers vote share log odds $V = \log(V_D/V_C)$ instead for two reasons. First, V_D ranges from 0 to 1, while V may take any natural number. If we assume both follow normal distribution, the V_D 's limited support is inappropriate. This is clear especially when we calculate the probability for the defender to win. For V , it is $\int_0^\infty \mathcal{N}(V)dV$, while, for V_D , it is $\int_0^1 \mathcal{N}(V_D)dV_D$. Second, effects of independent variables are larger when the current vote margin is smaller, namely, when election would be competitive without those effects. Thus, I use V instead of V_D .

Suppose that the vote at time t , V_t , be represented in the following way:

$$V_t = g_{V, t} + \varepsilon_{V, t}$$

$$g_{V, t} = \beta_0 + \beta_{QD}Q_{D, t} + \beta_{QC}Q_{C, t} + \beta_{MD}M_{D, t} + \beta_{MC}M_{C, t} + \beta_x x_t$$

where ε_V is an error term.

3.1.2 Normal Vote

Analysts usually control “normal vote,” that is, the vote the defender would have if all explanatory variables had no effect. Which measurement to use as the “normal vote” independent variable in x is, however, a controversial issue. An usual proxy is lagged vote Cox and Katz (2002); Gelman and King (1990); some may use presidential vote or vote for other electoral offices in the same district; others calculate their mean for a decade (Bond, Covington and Fleisher, 1985; Ansolabehere, Snyder and Stewart, 2000). I advocate for lagged vote, not just because it well explains the current election, but because the lagged dependent variable conveys unmeasured information.

I assume that the first order autoregressive (AR(1)) error process:

$$\varepsilon_{V, t} = \delta I_{t-1} \varepsilon_{V, t-1} + \epsilon_{V, t}$$

$$\epsilon_{V, t} \stackrel{iid}{\sim} N(0, \sigma_V^2)$$

where I_{t-1} is a binary variable to indicate whether the defender won or not at time $t - 1$:

$$I_{t-1} = \begin{cases} 1 & \text{if } V_{t-1} > 0 \\ -1 & \text{if } V_{t-1} \leq 0. \end{cases}$$

If a challenger won in the previous election, it becomes a defender in the current election and not $\varepsilon_{V, t-1}$ but $-\varepsilon_{V, t-1}$ shows its vote not explained by the model. That is why sign

is corrected by I_{t-1} . ϵ_V is unmeasured local shock at time t in the district. Examples are scandals, disasters, entry of a third party, redistricting, and so on. As always in AR(1) model, effects of past shocks are accumulated but discounted (forgotten) at the rate of $1 - \delta$ ($0 \leq \delta \leq 1$) election by election. Thus, δ is not just the coefficient of lagged vote but the discount parameter. I also assume that the current shock $\epsilon_{V,t}$ is unpredicted from (i.e. independent of) the past shocks $\epsilon_V(s < t)$ and their accumulation $\varepsilon_{V,t-1}$, but follows the same normal distribution.

Then,

$$\begin{aligned}
 V_t &= g_{V,t} + \varepsilon_{V,t} \\
 &= g_{V,t} + \delta I_{t-1} \varepsilon_{V,t-1} + \epsilon_{V,t} \\
 &= g_{V,t} + \delta I_{t-1} [V_{t-1} - g_{V,t-1}] + \epsilon_{V,t} \\
 &= \Delta(\delta) g_{V,t} + \delta I_{t-1} V_{t-1} + \epsilon_{V,t}
 \end{aligned}$$

where $\Delta(\delta)$ is a sign corrected partial difference operator,

$$\Delta(\delta) z_t = z_t - \delta I_{t-1} z_{t-1}$$

for any variable z_t . The parameters are estimated by regressing vote margin on sign corrected partially differenced covariates except the sign corrected previous vote margin which is not differenced.

Considering this way, $I_{t-1}[V_{t-1} - g_{V,t-1}]$ measures the normal vote: “the vote the defender would have if all explanatory variables had no effect”. For example, the previous vote margin which a challenger Democrat won in the previous open election has different meaning from that which an incumbent candidate of (defender) Democratic party won. Even if both are the same value, the former candidate is expected to be stronger than the latter. Thus, it is preferable to subtract covariates’ effect from the previous vote (see also Gowrisankaran,

Mitchell and Moro, 2004). And the coefficient of the previous vote, δ , shows the effect of the normal vote. This is reasonable substantially as well. For purpose of identification of δ , x does not include the previous vote.

Let the vote expectation function be

$$f_V(Q_D, Q_C, M_D, M_C, x) = \Delta(\delta)g_V(Q_D, Q_C, M_D, M_C, x|t) + \delta I_{t-1}V_{t-1}$$

where x includes lagged values of Q_D, Q_C, M_D, M_C and V . Then, the distribution of vote is

$$\begin{aligned} \mathcal{N}(v|q_D, q_C, m_D, m_C, x) &= \mathcal{N}(v|\text{mean} = \bar{V}, \text{variance} = \sigma_V^2) \\ \bar{V} &= f_V(Q_D = q_D, Q_C = q_C, M_D = m_D, M_C = m_C, x) \end{aligned}$$

where mean is conditioned on observed values.

3.2 Campaign Spending: M_D, C

Following Erikson and Palfrey (2000), we can approximate amount of campaign spending by a quadratic function of expected vote. Let the campaign spending expectation function of defender and that of challenger be quadratic of vote where no campaign money is spent:

$$f_{M_D, C}(Q_D, Q_C, x) = \gamma_{0D, C} + \gamma_{1D, C}\tilde{V}_{D, C}^2 + \gamma_{2D, C}\tilde{V}_{D, C}^2$$

where

$$\tilde{V}_D = -\tilde{V}_C = f_V(Q_D, Q_C, M_D = 0, M_C = 0, x)$$

In order to make γ 's comparable between defender and challenger, I let \tilde{V}_C be the negative of \tilde{V}_D . The literature on campaign effect almost agrees that a defender and a challenger collect and spend more money when an election seems to be competitive (namely, $\text{bar}V_{0D, C}$ is near zero): ($\gamma_{1D, C} = 0, \gamma_{2D, C} < 0$).

Then, the distribution of campaign spending is

$$\mathcal{N}(m_D|q_D, q_C, x) = \mathcal{N}(m_{D, C} | \text{mean} = \bar{M}_{D, C}, \text{variance} = \sigma_M^2)$$

$$\bar{M}_{D, C} = f_{MD, C}(Q_D = q_D, Q_C = q_C, x)$$

3.3 Quality of Candidate: $Q_{D, C}$

3.3.1 Expectation of Vote and Money

Let mean of $Q_{D, C}$'s Bernoulli distribution be denoted by $\bar{Q}_{D, C}(x)$:

$$\begin{aligned} Q_{D, C} &\sim \mathcal{B}(q_C|x) \\ &= \mathcal{B}(q_{D, C} | \text{mean} = \bar{Q}_{D, C}(x)) \end{aligned}$$

The value of $\bar{Q}_{D, C}(x)$ will be calculated as Bayesian Nash equilibria. Before specifying the function $\bar{Q}_{D, C}(x)$, I introduce some preparatory functions below.

Let f_{WD} be the conditional probability for the defender to win:

$$\begin{aligned} f_{WD}(Q_D, Q_C, M_D, M_C, x) &= Pr(V > 0 | Q_D, Q_C, M_D, M_C, x) \\ &= \int_0^\infty \mathcal{N}(v|\bar{V}, \sigma_V^2) dv \\ &= 1 - \Phi(0|\bar{V}, \sigma_V^2) \\ \bar{V} &= f_V(Q_D, Q_C, M_D, M_C, x) \end{aligned}$$

I denote the conditional probability for the challenger to win by

$$f_{WC} = 1 - f_{WD}.$$

Let $\lambda_{D, C}$ is benefit of seat. Conditioned on Q_D and Q_C , players can predict the expected amount of campaign money they will spend, $f_{MD, C}(Q_D, Q_C, x)$. Let electoral cost

be constant value plus campaign spending:

$$f_{KD, C}(Q_D, Q_C, x) = \kappa_{1D, C} + \kappa_{2D, C} f_{MD, C}(Q_D, Q_C, x)$$

Given Q_D, Q_C , the conditional expected utility is

$$\begin{aligned} & u_{D, C}(Q_D, Q_C, x) \\ &= f_{WD, C}(Q_D, Q_C, M_D = f_{MD}(Q_D, Q_C, x), M_C = f_{MC}(Q_D, Q_C, x), x) \times \lambda_{D, C} \\ &\quad - f_{KD, C}(Q_D, Q_C, x) \end{aligned}$$

3.3.2 Bayesian Nash Equilibria

Note that not all incumbent lawmakers leave House for electoral reasons (Box-Steffensmeier and Jones, 1997; Frantzich, 1978; Kiewiet and Zeng, 1993). Some have ambition for other offices such as senator or governor (Black, 1972; Brace, 1984; Copeland, 1989; Rohde, 1979). Some die. Others retire because they are too old, lose fun, or do not expect be promoted to the leadership (Brace, 1985; Groseclose and Krehbiel, 1994; Hall and Houweling, 1995; Hibbing, 1982; Theriault, 1998). Unfortunately, we are not sure of whether they leave Congress for electoral reason or not. Thus, I estimate it.

The challenger and the defender do not necessarily know each other's intention before deciding candidate's quality. Moreover, the challenger is not sure of whether the defender considers electoral prospect or not. Thus, I model this situation by static game of incomplete information. I assume that the defender has two types, electorally motivated party ($T_D = 1$) and non electorally motivated party ($T_D = 0$). A non electorally motivated type defender always field a non incumbent candidate. An electorally motivated defender decides candidate type by incumbent's utility. This includes primary defeat of incumbent. T_D is private information; the challenger does not know its value. The challenger is supposed to be always

electorally motivated ($T_C = 1$). $\tau_{D, C}$ is mean of $T_{D, C}$ ($\tau_C = 1$).

How does the electorally motivated defender ($T_D = 1$) decide to field which candidate, incumbent or non incumbent? When it fields incumbent ($Q_D = 1$), its expected utility is

$$\begin{aligned} u_D(Q_D = 1, x) &= u_D(Q_D = 1, Q_C = 1, x) \times \bar{Q}_C(x) \\ &+ u_D(Q_D = 1, Q_C = 0, x) \times (1 - \bar{Q}_C(x)) + \epsilon_{uD}. \end{aligned}$$

where ϵ_{uD} is an error term for the incumbent candidate of the defender. I assume that the incumbent will run if its expected utility is positive: When $u_D(Q_D = 1, x) > 0$, $Q_D(T_D = 1) = 1$. Otherwise, a non-incumbent will always run because entry in this situation maximizes its utility (Banks and Kiewiet, 1989): When $u_D(Q_D = 1, x) < 0$, $Q_D(T_D = 1) = 0$. When $u_D(Q_D = 1, x) = 0$, the incumbent may or may not run.

Non electorally motivated defender ($T_D = 0$) always fields nonincumbent: $Q_D(T_D = 0) = \bar{Q}_D(x|T_D = 0) = 0$.

How does the challenger decide to field which candidate, a strong candidate and a weak one? It is always electorally motivated: $T_C = \tau_C = 1$. When it fields a strong candidate ($Q_C = 1$), its expected utility is

$$\begin{aligned} u_C(Q_C = 1, x) &= \tau_D \times \left[u_C(Q_D = 1, Q_C = 1, x) \times \bar{Q}_D(x|T_D = 1) \right. \\ &\quad \left. + u_C(Q_D = 0, Q_C = 1, x)(1 - \bar{Q}_D(x|T_D = 1)) \right] \\ &+ (1 - \tau_D) \times u_C(Q_D = 0, Q_C = 1, x) + \epsilon_{uC} \end{aligned}$$

where

$$\begin{aligned} \bar{Q}_D(x) &= \bar{Q}_D(x|T_D = 1)\tau_D + \bar{Q}_D(x|T_D = 0)(1 - \tau_D) \\ &= \bar{Q}_D(x|T_D = 1)\tau_D \quad (\because \bar{Q}_D(x|T_D = 0) = 0). \end{aligned}$$

I also assume that a strong candidate will run if its expected utility is positive: when $u_C(Q_C = 1, x) > 0$, $Q_C = 1$. Otherwise, a weak candidate will always run (Banks and Kiewiet, 1989): when $u_C(Q_C = 1, x) < 0$, $Q_C = 0$. When $u_C(Q_C = 1, x) = u_C(Q_C = 0, x)$, a strong candidate may or may not run.

I assume that

$$\epsilon_{u_{D, C}} \stackrel{iid}{\sim} N(0, \sigma_{u_{D, C}}^2).$$

Let us denote

$$u_{DH} = u_D(Q_D = 1, Q_C = 0, x)$$

$$\Delta u_{DH} = (u_D(Q_D = 1, Q_C = 0, x) - u_D(Q_D = 1, Q_C = 1, x))$$

$$u_{CH} = u_C(Q_D = 0, Q_C = 1, x)$$

$$\Delta u_{CH} = u_C(Q_D = 0, Q_C = 1, x) - u_C(Q_D = 1, Q_C = 1, x)$$

Let S indicate which state of the world players are in. From the above argument, Bayesian Nash equilibria are:

- $S = 1$: When $u_D(Q_D = 1, x) \geq u_D(Q_D = 0, x)$ and $u_C(Q_C = 1, x) < u_C(Q_C = 0, x)$,

$$\bar{Q}_D(x|S = 1) = \tau_D, \quad \bar{Q}_C(x|S = 1) = 0$$

The probability that this situation occurs is

$$Pr(S = 1) = (\min(\Phi(u_{DH}), \Phi(\Delta u_{DH}))) \times (1 - \max(\Phi(u_{CH}), \Phi(\Delta u_{CH})))$$

where $\Phi(\cdot)$ is cumulative density function of the normal distribution whose mean is zero and variance is $\sigma_{u_{D, C}}^2$.

- $S = 2$: When $u_D(Q_D = 1, x) < u_D(Q_D = 0, x)$ and $u_C(Q_C = 1, x) \geq u_C(Q_C = 0, x)$,

$$\bar{Q}_D(x|S = 2) = 0, \quad \bar{Q}_C(x|S = 2) = 1$$

$$Pr(S = 2) = (1 - \max(\Phi(u_{DH}), \Phi(\Delta u_{DH}))) \times (\min(\Phi(u_{CH}), \Phi(\Delta u_{CH})))$$

- $S = 3$: When $u_D(Q_D = 1, x) > u_D(Q_D = 0, x)$ and $u_C(Q_C = 1, x) > u_C(Q_C = 0, x)$,

$$\bar{Q}_D(x|S = 3) = \tau_D, \quad \bar{Q}_C(x|S = 3) = 1$$

$$Pr(S = 3) = (\min(\Phi(u_{DH}), \Phi(\Delta u_{DH}))) \times (\min(\Phi(u_{CH}), \Phi(\Delta u_{CH})))$$

- $S = 4$: When $u_D(Q_D = 1, x) < u_D(Q_D = 0, x)$ and $u_C(Q_C = 1, x) < u_C(Q_C = 0, x)$,

$$\bar{Q}_D(x|S = 4) = 0, \quad \bar{Q}_C(x|S = 4) = 0$$

$$Pr(S = 4) = (1 - \max(\Phi(u_{DH}), \Phi(\Delta u_{DH}))) \times (1 - \max(\Phi(u_{CH}), \Phi(\Delta u_{CH})))$$

- $S = 5$: When $u_D(Q_D = 1, x) = u_D(Q_D = 0, x)$ and $u_C(Q_C = 1, x) = u_C(Q_C = 0, x)$,

$$\bar{Q}_D(x|S = 5) = \bar{Q}_D^*(x)$$

$$\bar{Q}_C(x|S = 5) = \bar{Q}_C^*(x)$$

$$Pr(S = 5) = |\Phi(u_{DH}) - \Phi(\Delta u_{DH})| \times |\Phi(u_{CH}) - \Phi(\Delta u_{CH})|$$

where

$$\bar{Q}_D^*(x) = \begin{cases} \frac{u_{CH}}{\tau_D \Delta u_{CH}} \tau_D & \text{if } \Delta u_{CH} \neq 0 \\ 0.5 \tau_D & \text{if } \Delta u_{CH} = 0 \end{cases}$$

$$\bar{Q}_C^*(x) = \begin{cases} \frac{u_{DH}}{\Delta u_{DH}} & \text{if } \Delta u_{DH} \neq 0 \\ 0.5 & \text{if } \Delta u_{DH} = 0 \end{cases}$$

Therefore, marginal mean of candidate quality dummy is

$$\begin{aligned}
\bar{Q}_{D, C}(x) &= \sum_{S=1}^5 \bar{Q}_{D, C}(x|S)Pr(S) \\
&= \min(\Phi(u_{D, CH}), \Phi(\Delta u_{D, CH})) \\
&\quad + |\Phi(u_{D, CH}) - \Phi(\Delta u_{D, CH})| \times \left(|\Phi(u_{C, DH}) - \Phi(\Delta u_{C, DH})| \right. \\
&\quad \left. + E(\bar{Q}_{D, C}^*(x)|S = 5) \right)
\end{aligned}$$

where

$$E(\bar{Q}_{D, C}^*(x)|S = 5) = \int_0^{\Delta u_{D, CH}} \frac{u_{D, CH}}{\Delta u_{D, CH}} N(u_{D, CH} | \text{mean} = E(u_{D, CH}))$$

which I approximate by

$$\frac{1}{2} \left(\frac{0}{\Delta u_{D, CH}} + \frac{\Delta u_{D, CH}}{\Delta u_{D, CH}} \right) = 0.5.$$

3.4 Estimation

So far, I have specified conditional distributions of five endogenous variables. Multiplying them, their joint distribution is obtained. In order to identify λ s and κ s, $\sigma_{u_{D, C}}^2$ is supposed to be 1. For fear of unit root problem, I also assume that δ is equal to 1. It is substantially reasonable as well because previous electoral shocks are thought to affect the following elections for a while. Then, parameters β s, γ s, σ^2 s, τ_D , λ s and κ s are estimated by maximizing likelihood function.

4 Monte Carlo Simulation

I perform Monte Carlo simulation to study how much simultaneity bias contaminates the conventional estimator but not my estimator. A party indicator P is used as the only exogenous variable X . $P = 1$ means Democrat defender and $P = -1$ means Republican defender. Parameters are set as follows: $\sigma_V^2 = 0.5, \sigma_{MD}^2 = 1.2, \sigma_{MC}^2 = 0.8, \tau = 0.9, \gamma_D = (9.5, 0.1, -1), \gamma_C = (8.5, -0.1, -1), \kappa_D = \kappa_C = (1.5, 0.1,), \lambda_D = \lambda_C = 3, \beta_{QD} = 0.2, \beta_{QC} = -0.2, \beta_{MD} = 0.2, \beta_{MC} = -0.2, \beta_P = 0.3, \text{Constant} = 0.5$. I randomly produce all lagged values $(V_{t-1}, Q_{D,t-1}, Q_{C,t-1}, M_{D,t-1}, M_{C,t-1}, P_{t-1})$ only once. The value of P_t is calculated as $I_{t-1}P_{t-1}$. Then, I make 50 sets of endogenous variables $(V_t, Q_{D,t}, Q_{C,t}, M_{D,t}, M_{C,t})$.

I examine two alternative models. First, the most straight-forward model is to regress the current vote on the endogenous variables and the sign corrected lagged dependent variable. I call this “lagged vote model”.

$$\begin{aligned}
 V_t &= \beta_0 + \beta_{QD}Q_{D,t} + \beta_{QC}Q_{C,t} + \beta_{MD}M_{D,t} + \beta_{MC}M_{C,t} + \beta_P P_t \\
 &\quad + \delta I_{t-1}V_{t-1} + \epsilon_{V,t} \\
 \epsilon_{V,t} &\stackrel{iid}{\sim} N(0, \sigma_V^2)
 \end{aligned}$$

But this model does not remove previous exogenous variables’ effects from lagged vote. The second easy model, “normal vote model”, is to include all sign corrected lagged variables (since sign corrected $I_{t-1}P_{t-1}$ is equal to P , I drop it).

$$\begin{aligned}
 V_t &= \beta_0 + \beta_{QD1}Q_{D,t} + \beta_{QC1}Q_{C,t} + \beta_{MD1}M_{D,t} + \beta_{MC1}M_{C,t} + \beta_{P1}P_t \\
 &\quad + \delta I_{t-1}V_{t-1} + \beta_{QD2}I_{t-1}Q_{D,t-1} + \beta_{QC2}I_{t-1}Q_{C,t-1} \\
 &\quad + \beta_{MD2}I_{t-1}M_{D,t-1} + \beta_{MC2}I_{t-1}M_{C,t-1} + \epsilon_{V,t}
 \end{aligned}$$

Table 1 compares means and standard deviances of 50 ML estimates of some parameters

by the three models. It is natural that my estimator is unbiased and efficient, because the data generation process is exactly the same as the model assumes.² An important result is that the lagged vote model overestimates incumbency advantage (β_{QD}) as suspected. Moreover, it underestimates defender’s campaign spending effect (β_{MD}) and overestimate challenger’s (β_{MC}), which also supports the common concern. Besides, this model estimates high quality challenger’s effect (β_{QC}) to be larger than it is. For all of them, standard deviances of the lagged vote model are larger than those of my model. The normal vote model is better than the lagged vote model. Naturally, the coefficients of the lagged variables are almost close to the negative values of the coefficients of the current variables. But even the normal vote model is as half as efficient as my model and some parameters (Q_C , P , and the constant) are biased. Therefore, if there is simultaneity problem in the real electoral data, these two alternative models will produce biased and inefficient estimates as often expected and my estimator is preferable.

	True Value	My Model		Lagged Vote Model		Normal Vote Model	
		Mean	SD	Mean	SD	Mean	SD
Q_D	0.2	0.198	0.018	0.258	0.040	0.197	0.033
Q_C	-0.2	-0.202	0.018	-0.590	0.074	-0.185	0.066
M_D	0.2	0.201	0.008	0.104	0.015	0.199	0.012
M_C	-0.2	-0.200	0.006	-0.418	0.013	-0.201	0.016
Constant	0.5	0.494	0.074	2.946	0.117	0.515	0.145
P	0.3	0.003	0.019	0.001	0.020	-0.000	0.016
V_{t-1}				0.292	0.047	0.995	0.048
$Q_{D,t-1}$						-0.226	0.044
$Q_{C,t-1}$						0.200	0.039
$M_{D,t-1}$						-0.227	0.015
$M_{C,t-1}$						0.174	0.014

Table 1: Results of Monte Carlo Simulation

²Estimates of the other parameters except λ_D and κ_D are also unbiased and efficient. The true values are within two standard errors from the estimates. Better estimation of λ_D and κ_D is one of future agendas.

5 Empirical Analysis

5.1 Data

I use the U.S. House election data, 1972 to 2004, made by Gary Jacobson.³ I delete observations which include any missing value or do not have one major party defender candidate and one challenger candidate (mostly because of redistriction). The number of observations is 4927.

Endogenous variables are:

- *Vote* (V): Log odds of the defender's two-party vote share.
- *Defender's Quality* (Q_D): A dummy variable of incumbent candidate.
- *Challenger's Quality* (Q_C): A dummy variable which indicates whether the candidate has held elective office or not.
- *Defender's Spending* (M_D, C): Logarithm of defender's expenditures plus one.
- *Challenger's Spending* (M_D, C): Logarithm of challenger's expenditures plus one.

Exogenous Variables (x) are:

- *Democrat.19XX*: 16 dummy variables which indicate whether the defender party is Democrat or not in year 19XX (1974 to 2004). All observations in 1972 are dropped because lagged variables are not available. These variables control each year's national tide.
- *Constant*.

³Gary Jacobson kindly gave me his data. I appreciate him.

5.2 Results

5.2.1 Effects on Vote (β)

To make clear how different my model is from previous ones, I compare my estimates with those of the lagged vote model and the normal vote model.⁴ Since raw estimates of parameters are difficult to interpret, I will demonstrate effects of endogenous variables and exogenous ones by simulation (King, Tomz and Wittenberg, 2000).

Table 2 reports first differences of vote. The method to calculate them is as follows. Using ML estimates by my model, I calculate the mean of empirical normal votes, $v_N = \log \text{odds}(0.739)$. Using it and my campaign spending expectation function, I calculate the baseline spending levels: $m_{DN} = f_{MD}(Q_D = 0, Q_C = 0, v_N) = \log(111, 922)$ and $m_{CN} = f_{MC}(Q_D = 0, Q_C = 0, v_N) = \log(128, 935)$. I add their effects to v_N and obtain the baseline vote value, $v_B = v_N + (\beta_{MD,my\ model}m_{DN}) + (\beta_{MC,my\ model}m_{CN}) = \log \text{odds}(0.614)$.

	Mean	SD	Mean	SD	Mean	SD
Defender's Quality	4.7	0.2	5.4	0.3	5.9	0.3
Challenger's Quality	-2.8	0.2	-3.3	0.3	-3.2	0.3
Defender's Spending	-1.8	0.1	-2.1	0.1	-1.9	0.1
Challenger's Spending	-1.1	0.1	-3.0	0.1	-3.1	0.1

Table 2: First Differences: The Effects of Endogenous Variables on Vote

I make 1,000 samples of parameters using ML estimates and covariance matrix. For each sample, I calculate the values of vote which the three models estimates where quality of candidate changes from zero to one or campaign spending increases from the baseline by one standard deviance error estimated by my model ($\hat{\sigma}_{MD,my\ model}^2 = 2.49, \hat{\sigma}_{MC,my\ model}^2 = 2.80$). For example, in the case of incumbent candidate, I calculate $v_B + (\beta_{QD,one\ model} \times 1)$. In the case of challenger's spending, $v_B + (\beta_{QC,one\ model} \hat{\sigma}_{MC,my\ model}^2)$. Finally, I calculate

⁴For optimization of my model, I use `rgenoud` library in a statistics software R (Sekhon and Mebane, 1998). In Monte Carlo simulation of the previous section, I used `optim` command.

mean and standard deviances. I do not take into consideration random noise, ϵ_V . Thus, these are average treatment effects.

According to Table 2, compared to the other two models, my model correctly underestimates effects of defender's candidate quality by nearly one percentage point and that of challengers' campaign spending by two percentage points but overestimates that of defender's campaign spending slightly, taking into consideration of simultaneity bias (though, unfortunately, even my model estimates defender's campaign spending effect to be negative, much less the other two). Figure 1 makes clear difference of incumbency advantage estimates among the three models. It illustrates the three distinct densities of estimates by the three models.

5.2.2 Effects on Campaign Spending (γ)

Figure 2 displays that, the more competitive the normal vote is, the more campaign money each candidate spend. Also, entry of strong challenger candidate adds fuel to the race and makes more finance necessary, while incumbent's reelection bid makes the election lopsided and depresses electoral expenditures by both parties. All these results are as expected.

5.2.3 Effects on Candidate Quality (λ, κ, τ)

Figure 3 shows the probabilities for high quality candidate to run depending on normal vote size. It is clear that, as normal vote becomes smaller, an incumbent hesitates to enter the race and a strong challenger candidate is more willing to run. This is why simultaneity bias occurs. The incumbent's probability hits the ceiling of $\hat{\tau} = 0.892$. This means that almost 10 % incumbents quit Congress for non electoral reasons.

Some works show that lagged vote affects incumbent's decision to run, though few take expectation of vote into consideration of candidate quality and explain both simultaneously. This is my departure from previous studies.

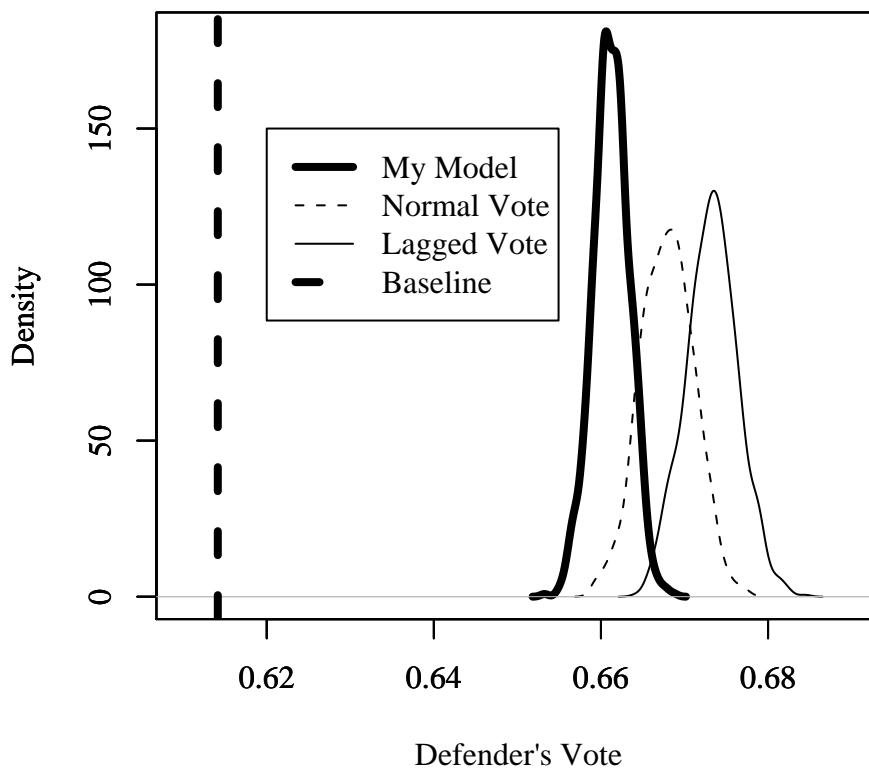


Figure 1: Incumbency Advantage

6 Conclusion

This paper proposes a solution to parametric dependence of simultaneity bias of incumbency advantage and campaign spending. The method is to model each endogenous variables by the others (or their expectation) by making much of theories of electoral politics and maximize the total likelihood derived from the joint distribution of vote, both parties' candidate qualities and campaign spending. I show superiority of my model compared to the conventional estimators by Monte Carlo simulation. Empirical application of this model to the recent U.S. House election data demonstrates that incumbency advantage is smaller than

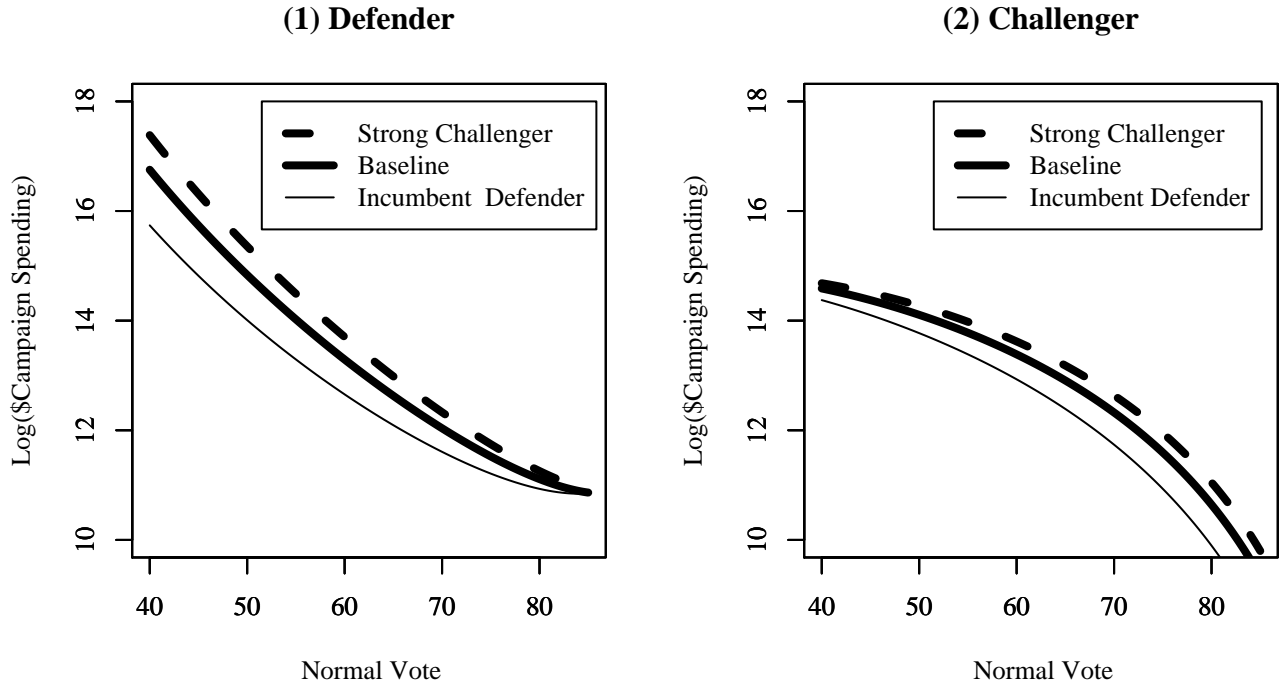


Figure 2: Campaign Spending and Normal Vote

previously shown and that entry of incumbent and strong challenger is motivated by electoral prospect.

I also intend to contribute to electoral studies on the two points. First, my model subtracts effects of lagged variables from the lagged vote to obtain the normal vote, because substantial meaning of lagged vote differs depending on how it was fought. Second, the present paper derives Bayesian Nash equilibria of candidate choice decision by both parties and plugs this game theoretic solutions into the statistical model.

It goes without saying that my model can be applied to any single member district election fought by the two major parties beyond the U.S. Moreover, you can use it in analyzing mixed proportional representation (PR) electoral system. Ferrara, Herron and Nishikawa (2005) argue that a party which fields a candidate in a single member district (SMD) has bonus

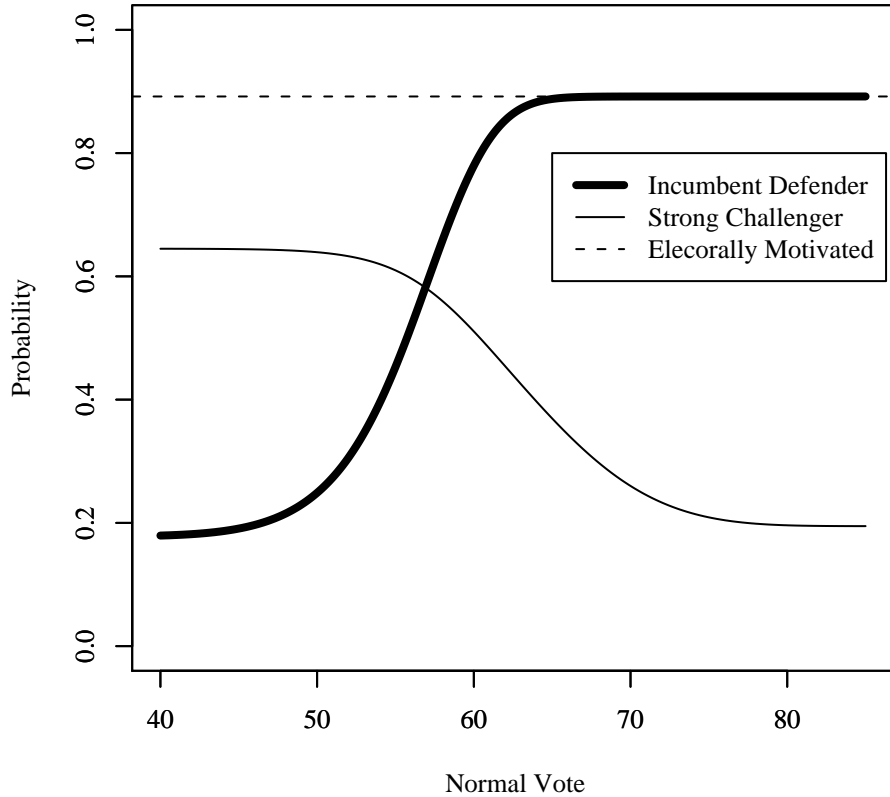


Figure 3: Probabilities for High Quality Candidate to Run

votes in PR tier in that SMD. If you take $Q_{D, C}$ as a dummy of SMD candidate and V as PR vote share log odds and collapse parties into two major blocs, you can use my model.

As a matter of course, many future agendas remain to be solved. One is solution to stochastic dependence. My model can (and should) be combined with stochastic dependence model. But note that, while my model analyze not only vote but also candidate quality and campaign spending at the same time, natural experiment method can not because it tries to control all variables except one. Another issue is my implicit assumption that parties and candidates know the same correct distribution of vote, $fV(\cdot)$ and σ_V^2 . In reality, they should

have different information and, therefore, different expectation of vote. Moreover, variances may vary with the electoral situation. For example, non incumbent entry or redistribution will deteriorate uncertainty. Third, discount rate of δ is also a quantity of interest and its estimation is promising. Error process should be examined more intensively. These are open to future works.

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