Survival Analysis of Systematically Dependent Competing Risks: An Application to the U.S. Congressional Careers *

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Abstract

Competing risks model of survival analysis studies whether, when and why (or how) an event happens on a subject. Sometimes, these risks are dependent on each other. For example, when lawmakers expect they would lose election, they will strategically choose to retire in order to avoid cost of electoral campaign. The systematically dependent competing risks model of survival analysis I proposed in this paper enables us to estimate more than two risks, which is almost impossible for the currently popular frailty model. An application to the U.S. Congressional careers is demonstrated.

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INTRODUCTION

Survival analysis is one of tools which are getting popular in the field of political science. It shows when and whether an event happens. Besides, competing risks model makes clear why or how it happens. Most studies, however, (inadvertently) assume that competing risks are independent, while this assumption is sometimes not reasonable.

The running example in the paper is the U.S. Congressional career. Legislators exit the legislature for various reasons. In the U.S. Congressional case, Box-Steffensmeier and Jones (1997, 2004) identify four: retire, ambition (running for other public offices such as senator and governor), electoral termination at primary or general. These four events are not necessarily independent. For example, when lawmakers expect they would lose election even if they ran again, they will strategically choose to retire in order to to avoid cost of electoral campaign and losing face.

In order to address dependent competing risks, previous works use frailty model, though it is extremely difficult to model more than two risks. I propose a systematically dependent competing risks model of survival analysis, which can model any number of risks. In addition, I tailor the model so that it captures the data generation process of legislators' exit more appropriately, considering non-random censoring and nested risks structure. Political science is full of cases whose data generation process is well represented by dependent competing risks model. To name only a few, cabinet resolution, war or peace duration, and survival of administrative organization are promising applications.

The remainder of the paper is organized as follows. The first section develops a systematically dependent competing risks model of survival analysis in general terms and then presents the specific model for Congressional careers. The next section applies the model to U.S. Congressional career data. Finally, I conclude.

1 MODEL

For studying timing and type of end of duration such as legislators' political life, survival analysis is appropriate. To examine dependence between risks, a dependent competing risks model is necessary. First, I introduce a general model of competing risks and propose a systematically dependent competing risks model. I then specify a custom version of the model so that it fits Congressional careers.

1.1 A General Model of Survival Analysis of Dependent Competing Risks

1.1.1 Competing Risks

I consider the case where there are three risks ($r \in \{1, 2, 3\}$, for example, 1=not run (in order to run for Senator or Governor (ambition) or just to retire), 2=lose at primary election, 3=lose at generel election), though it is easy to model any number of risks. y indicates type of exit or censoring. When an event E_r due to risk r occurs, $y = r$. If duration of a subject is right censored without any event (e.g., reelected), $y = 0$. y_r is a dummy variable which is one when an event E_r happens ($y = r$) and otherwise zero. $y_1 + y_2 + y_3 \in \{0, 1\}$ because only one event due to one risk is observed. T_r is the potential time when E_r would happen if other types of events did not happen and a subject (e.g., a legislator) were not censored. T is the observed time when a subject exits ($T = min(T_r)$) or is censored.

Once any event occurs on a subject, it exits (from all risk sets) and no more event happens.[1] The probability density that an event E_r occurs at time t given that a subject "survives" at that time ($T_r \geq t$), $p(y_r = 1, T_r = t|T_r \geq t) \geq 0$, is called hazard and denoted by $h_r(t)$. Let $h(t) = p[(y_1 + y_2 + y_3) = 1, min(T_r) = t|min(T_r) \geq t]$, then,

$$h(t) = B(y_1|h_1(t)) \times B(y_2|h_2(t|y_1)) \times B(y_3|h_3(t|y_1, y_2)) \tag{1}$$

[1] To be accurate, defeated incumbents may come back to Congress. I discuss repeated events in another paper (Fukumoto, 2005). See Box-Steffensmeier and Jones (2002) and Box-Steffensmeier and Jones (2004)

where $B(y|\theta)$ is the Bernoulli distribution with mean $0 < \theta < 1$, $B(y|\theta) = \theta^y(1-\theta)^{1-y}$.

1.1.2 Non-Random Censoring

If censoring of every risk r duration by any other type of event E_s ($y = s \neq r$) or non event ($y = 0$) is uninformative (i.e., at random or statistically independent of $h_s(t)$), then, $h_2(t|y_1) = h_2(t)$, $h_3(t|y_1, y_2) = h_3(t)$ and the conditional probabilities in Eq. (1) are reduced to marginal ones.

$$h(t) = B(y_1|h_1(t)) \times B(y_2|h_2(t)) \times B(y_3|h_3(t)) \tag{2}$$

Most of the previous studies (sometimes implicitly) assume random censoring and use this equation. In not a few cases, however, this assumption of random censoring is dubious. For example, (in continuous time model) retirement may be more likely to occur when electoral loss is more prospective (even if conditioned on covariates). In this case, censoring electoral defeat risk duration by retirement (E_1) is not at random but positively correlated with electoral defeat hazard ($h_3(t)$):

$$\frac{\partial h_3(t)}{\partial y_1} > 0 \tag{3}$$

If they are negatively correlated, the left hand side of Eq. (3) is less than zero. If they are not lineally correlated, it is equal to zero. Therefore, when censoring is not independent of other types of event occurrence, Eq. (1) should be used instead of Eq. (2).

1.1.3 A Systematically Dependent Competing Risks Model

In the present subsection, I model hazard for risk r as proportional to baseline hazard for risk r, $h_{r(0)}(t)$, and make the rate a function of linear predictor of covariates x_r (proportional hazard model): $h_r(t|x_r) = \exp(x_r\beta_r)h_{r(0)}(t)$.[2] The most common way to model dependent competing risks is frailty model, which is a kind of random effect model (Box-Steffensmeier and Jones, 2004; Gordon, 2002):

$$\begin{aligned} h_r(t|x_r) &= \exp(x_r\beta_r + \nu_r)h_{r(0)}(t) \\ h_s(t|x_s) &= \exp(x_s\beta_s + \nu_s)h_{s(0)}(t) \\ (\nu_r, \nu_s) &= \text{Multivariate Normal}\big((0,0), \Omega\big) \end{aligned}$$

where νs are random variables independent of covariates and are called frailty, and Ω is a variance-covariance matrix. To estimate, one needs to integrate out frailties through Markov Chain Monte Carlo or numerical integration. In the case of more than two risks, however, it is very difficult to identify estimates (Gordon, 2002). In addition, frailty model takes into consideration stochastic dependence only, not systematic dependence.

Instead, I propose a systematically dependent competing risks model, where the hazard for one risk is conditional on the same linear predictor of covariates for other hazards and, unlike stochastically dependent models, is conditionally independent of other hazards. For two risks,

$$\begin{aligned} h_r(t|x_r) &= \exp(x_r\beta_r)h_{r(0)}(t) \\ h_s(t|x_s, x_r) &= \exp(x_s\beta_s + \gamma_{rs}(x_r\beta_r))h_{s(0)}(t) \end{aligned} \tag{4}$$

where γ_{rs} is dependence parameter.[3] The more variables in x_r are contained in x_s, the more

[2] If you do not employ proportional hazard model, you only have to replace β_r with $\beta_r(t)$. I discuss baseline hazard shortly.

[3] To put it differently, this is a constrained model as follows:

$$\begin{aligned} h_r(t|x_r) &= \exp(x_r\beta_r)h_{r(0)}(t) \\ h_s(t|x_s, x_r) &= \exp(x_s\beta_s + x_r\beta_{rs})h_{s(0)}(t) \\ \beta_{rs} &= \gamma_{rs}\beta_r \end{aligned}$$

severe multicollinearity becomes a problem (not vice versa). Also, for every pair of hazards, at most one hazard may have dependence parameter for the same reason. These limitations, though, pay because it forces us to consider the directions of causality carefully. In contrast to the standard frailty model, this model can handle more than two risks and is easier and faster to estimate by maximum likelihood.

1.2 The Specific Model for Congressional Careers

So far, I have introduced dependent competing risks models in general terms. In this subsection, I tailor the model so that it captures the data generation process of Congressional careers more appropriately.

1.2.1 Discrete Time

I use a discrete time model, rather than a continuous one. There are a few reasons. First, time here is not the exact duration of legislators' political life in months or days but the number of terms they serve in the legislature. Seniority, for example, is determined by the number of terms a member serves. Second, it is easier and more flexible to incorporate time varying covariates such as age and electoral strength into discrete time model Box-Steffensmeier and Jones (2004). Third, a discrete time model enables me to use the inverse logistic link with which more political scientists are familiar (Beck, Katz and Tucker, 1998). In this case, the hazard is not probability density which has only a lower bound of zero but a true probability that ranges from zero to one.

1.2.2 Nested Competing Risks

In the present case, a subject becomes at one type of risk after it ceases to be at the other types of risks. I call this structure nested competing risks. Only after lawmakers decide to run for an election ($y_1 = 0$ is confirmed), they are at electoral risk. Therefore, in the case where they choose not to run ($y_1 = 1$), there is no possibility of electoral loss at neither primary nor general: $h_2(t|y_1 = 1) = h_3(t|y_1 = 1) = 0$ (I take into consideration how electoral prospect affects the probability not to run). Similarly, they become at risk of general election failure only if they survive primary. By contrast, when incunmbents are defeated at primary ($y_2 = 1$), they never lose general: $h_3(t|y_1 = 0, y_2 = 1) = 0$. Therefore, I abuse notation and redefine $h_2(t)$ and $h_3(t)$ as $h_2(t|y_1 = 0)$ and $h_3(t|y_1 = 0, y_2 = 0)$, respectively.[4] Hence, according to Eq. (1), marginal hazard, $p(y_r = 1, T_r = t|T_r \geq t)$, is

$$\begin{aligned}
h(t, y = 1) &= h_1(t) \\
h(t, y = 2) &= [1 - h_1(t)] \times h_2(t) \\
h(t, y = 3) &= [1 - h_1(t)] \times [1 - h_2(t)] \times h_3(t)
\end{aligned}$$

In the multinomial logit model, these are modeled with covariates. Quantities of my interest are, however, not that but conditional hazard, $h_r(t) = p(y_r = 1, T_r = t|T_r \geq t, y_s = 0 \forall s < r)$, which I parameterize as the binary logit model:

$$h_r(t) = \frac{1}{1 + \exp[-(g_r(x_t) + h_{r(0)}(t))]}$$

where $h_{r(0)}(t)$ is log odds of baseline hazard, $g_r(x_t)$ is a proportinality function and x_t is time varying covariates. Here, odds of hazard, not hazard itself, is made proportional to odds of baseline hazard.

[4] A general remark is in order. If nested risks are independent conditioned on previous events, that is, if covariates of any hazard never share those of other hazard in Eq.(4), each hazard is parametrically independent and possible to estimate by nested (multinomial) logistic regression, using those observations only which are at the risk (e.g., for primary loss hazard, only analyzing legislators who run for primary). Box-Steffensmeier and Jones (1997) take this strategy, though without mentioning non-random censoring and insufficiently in the sense that they do not omit primary election losers when they study general election defeat. In their latest work, Box-Steffensmeier and Jones (2004) do not even introduce this nested logistic regression. That is regrettable because nested (multinomial) logistic regression is easily available through any canned software (e.g. estimate separate regressions by selecting observations which are at that risk) and is appropriate only if scholars can assume that competing risks are nested and parametrically independent.

1.2.3 Within-Nest Competing Risks

To be accurate, choice of not running (E_1) is composed of two events: ambition ($y = 1a$) and retirement ($y = 1b$). Let the hazard of ambition and that of retirement denoted by $h_{1a}(t)$ and $h_{1b}(t)$, respectively. Though Box-Steffensmeier and Jones (2004, 168-75) introduce multiple binomial logistic regressions and single multinomial logistic regression using all observations for independent competing risks, the former is not appropriate. Multiple binary logistic regressions do not constrain the data generation process so that only one event is observed. Thus, I use single multinomial logistic regression. I parameterize the two conditionally independent risks as follows.

$$
\begin{aligned}
h(t, y = 1a) &= h_{1a}(t) \\
&= \frac{\exp[g_{1a}(x_t) + h_{1a(0)}(t)]}{1 + \exp[g_{1a}(x_t) + h_{1a(0)}(t)] + \exp[g_{1b}(x_t) + h_{1b(0)}(t)]} \\
h(t, y = 1b) &= h_{1b}(t) \\
&= \frac{\exp[g_{1b}(x_t) + h_{1b(0)}(t)]}{1 + \exp[g_{1a}(x_t) + h_{1a(0)}(t)] + \exp[g_{1b}(x_t) + h_{1b(0)}(t)]} \\
h_1(t) &= h_{1a}(t) + h_{1b}(t) \qquad (\because \text{the two risks are conditionally independent})
\end{aligned}
$$

1.2.4 Dependence between Risks

I expect general election hazard (h_3) increases primary one (h_2, $\gamma_{32} > 0$) and facilitates retirement risk (h_{1b}, $\gamma_{31b} > 0$). It has nothing to do with ambitious leave (h_{1a}, $\gamma_{31a} = 0$). Lawmakers with high ambition hazard may be less likely to retire ($\gamma_{1a1b} < 0$). Since primary hazard is correlated to general hazard and I want to avoid multicollinearity problem, I do not model dependence between primary hazard and ambition as well as retirement. Therefore, a proportionality function for every risk is as follows (I omit time subscript t from x's):

$$
\begin{aligned}
g_{1a}(x) &= x_{1a}\beta_{1a} + \gamma_{31a}(x_3\beta_3) \\
g_{1b}(x) &= x_{1b}\beta_{1b} + \gamma_{31b}(x_3\beta_3) + \gamma_{1a1b}(x_{1a}\beta_{1a}) \\
g_2(x) &= x_2\beta_2 + \gamma_{32}(x_3\beta_3) \\
g_3(x) &= x_3\beta_3
\end{aligned}
$$

where x_r is covariates for risk r and $x = x_{1a} \cup x_{1b} \cup x_2 \cup x_3$.

1.2.5 Baseline Hazard: The Log-Odds-Quadratic Time Dependence

In discrete time model, if one uses dummy variables for every term (Cox model), one does not have to assume any shape of baseline hazard. Since events may be relatively rare for some terms, however, estimates of these dummies are less efficient. Besides, my primary interest does not lie in the exact shape of baseline hazard but in whether it increases or decreases as a legislator serves for more terms, or irn the quadratic relationship between term and hazard. For example, freshmen Representatives are not powerful enough to run for Senator or Governor, while seniors accumulate too much stake in Congress to exchange even for these higher offices. Then, it suffices to model log odds of baseline hazard as the quadratic function of the number of terms:

$$
h_{r(0)}(t) = \alpha_r + \tau_r t + \upsilon_r t^2
$$

where α_r is a constant term, τ_r is a linear time dependence parameter and υ_r is a quadratic time dependence parameter.

1.2.6 Likelihood

Generally speaking, in a discrete time model, an observation of a legislator i at term t appears in the dataset only if the legislator survives the previous terms. I denote the likelihood of legislator i's exit type y at term t by $\mathscr{L}_i(y|t)$. When any event occurs,

$$\begin{aligned}\mathscr{L}_i(y > 0|t) &= p[(y_{1i} + y_{2i} + y_{3i}) = 1, min(T_{ri}) = t|min(T_{ri}) \geq t] \\ &= h(t)_i\end{aligned}$$

By contrast, when no event occurs during the term, the lawmaker is reelected and survives and the term is right censored. Then,[5]

$$\begin{aligned}\mathscr{L}_i(y = 0|t) &= 1 - \sum_{r=1}^{3}[h(t|y_r = 1)_i] \\ &= [1 - h_1(t)_i] \times [1 - h_2(t)_i] \times [1 - h_3(t)_i]\end{aligned}$$

Suppose a legislator i is observed to t_i and there are n legislators. Then, total likelihood $\mathscr{L}$ is

$$\mathscr{L} = \prod_{i=1}^{n}\prod_{t=1}^{t_i}\mathscr{L}_i(y|t)$$

[5] This is equivalent to a conditional survival function.

2 AN APPLICATION TO THE U.S. CONGRESSIONAL CAREERS

2.1 Data

In order to illustrate what this model reveals, I apply it to the U.S. Congressional career data Box-Steffensmeier and Jones (2004) use.[6] The original authors assume that competeing risks are independent and employ separate binomial logit and multinomial logit. Even if they introduce the fraility model for dependent competeing risks, they do not use it because there are more than two (actually, four) risks which the model can not handle currently. Box-Steffensmeier and Jones (1997) utilize nested multinomial logistic regression so that they take into considertaion nested risk, though even this model can not analyze dependence between competeing risks. In contrast to the original author's models, this paper can shed new light on dependent competeing risks. Another difference is baseline hazard: theirs is logarithm of duration, while mine is the quadratic form.

The data is comprised of information pertaining to the careeer path for every House member elected in each freshman class from 1950 to 1976. Each member of the House was tracked from his or her first reelection bid until the last term served in office. Dependent variables are exit type (retire, ambition, primary, general and reelection) and the number of terms they serve up to the observation. Independent variabvles are:

- Republican (the original name is Party)
- Redistrict: whether or not the incumbent's district was substantially redistrcited
- Scandal: whether or not the incumbent was involved in scandal

[6] Their data and code are downloaded from `http://www.u.arizona.edu/ bsjones/eventhistory.html`. I appreciate the authors for the data.

- Open Gub.: whether or not there was an open gubernatorial seat available during the election cycle
- Open Sen.: whether or not there was an open U.S. Senatorial seat available during the election cycle
- Leadership: whether or not the incumbent had a leadership position in the House
- Age
- Prior Margin: the percentage of votes the incumbent (or his or her party) received in his or her previpous election

All of them are dummies except for the last two. I listwise delete observations which have missing values to get 5320 observations.

In order to avoid multicollinearity, I include redistrict, scandal, leadership and past electoral record variables for general election risk only, because these variables have direct effects on general election hazard, through which they may have indirect effects on other events.

2.2 Results

Table 1 reports the results.[7] One of new findings is that the direct effect of age on retirement risk is significantly differentr from zero even if one takes into account its indirect effect through general election risk. Since this model is too complicated to interpret these parameter estimates, I illustrate some of them by simulation.

Table 2 shows (expected) first differences of hazards by risk. I set all dummies at zero (for dummies) or their mean values (for continuous variables, 51.4 for age and 35.6 for prior margin), which I call reference values, and calculate benchmark hazard for every risk by using point estimates of parameters. Then, for every covariates, I add one to dummies or a

[7] I write codes and estimate parameters of the model on a statistics software `R`.

	Generel			Primary			Retire			Ambition		
	Est.	S.D.		Est.	S.D.		Est.	S.D.		Est.	S.D.	
Republican	−0.18	0.13		−0.22	0.26		0.19	0.15		0.30	0.14	*
Redistrict	1.65	0.31	**				0.07	0.01	**			
Scandal	3.40	0.40	**									
Open Gub.	0.12	0.16								0.47	0.16	**
Open Sen.	−0.25	0.21								0.98	0.16	**
Leadership	−1.19	0.58	*									
Age	0.04	0.01	**	0.02	0.02					−0.06	0.01	**
Prior Margin	−0.06	0.01	**									
Constant	−3.24	0.39	**	−5.86	0.81	**	−8.17	0.50	**	−1.98	0.47	**
Time	−0.15	0.06	*	0.13	0.12		0.28	0.06	**	0.6	0.11	**
Time^2	0.01	0.01		−0.01	0.01		−0.01	0	**	−0.05	0.01	**
γ General				0.29	0.11	**	0.18	0.05	**	0.01	0.05	
γ Ambition							0.06	0.19				

Table 1: A Systematically Dependent Competing Risks Model of the U.S. Congressional Career Paths

standard deviance to continuous variables (9.5 for age and 28.0 for prior margin) and calculate hazards. After one subtracts benchmark values from these hazards, one gets (expected) first differences.

	General	Primary	Retire	Ambition
Republican	−0.6	−0.2	0.3	0.4
Redistrict	13.8	0.6	0.5	0.0
Scandal	51.5	1.7	1.2	0.0
Open Gub.	0.5	0.0	0.1	0.7
Open Sen.	−0.8	−0.1	0.0	1.8
Leadership	−2.8	−0.3	−0.3	0.0
Age	1.9	0.4	1.6	−0.5
Prior Margin	−3.2	−0.4	−0.3	0.0
Benchmark	4.0	1.1	1.4	1.1

Table 2: First Differences of Hazards by Covariates

Quantities of most interest are between-hazards dependence parameters, γ's. Only the dependence of primary on general (γ_{32}) and that of retirement (γ_{31b}) significantly different from zero (positive).[8] Figure 1(1) shows how primary and retirement hazards at the first term increase as the general election hazard rises. I fix all covariates at the reference values mentioned above except prior electoral margin, which I move from its empirical minimum (0) to maximum (100). Thus, primary and retirement hazards change only because of general election risk's increment. According to this figure, the more likely legislators are to fail to be reelected at general (larger h_3), the more likely they are to be defeated at primary (larger h_2) or to retire (larger h_{1b}).

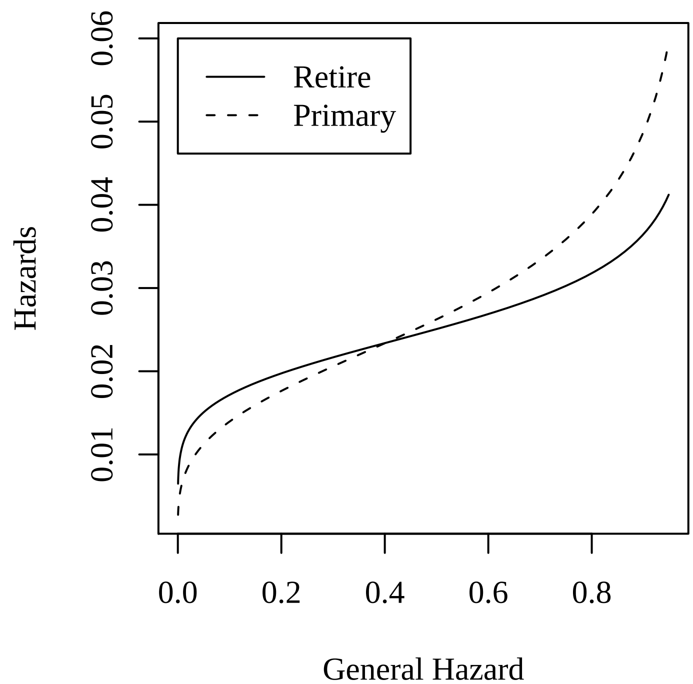

Figure 1: Dependence between Competing Risks

Figure 2 depicts baseline hazards along electoral terms when covariates are set at the

[8]Independence between ambition and retirement can not be rejcted. This result supports the original analysis (Box-Steffensmeier and Jones, 2004)

reference values. As expected, ambition hazard reaches its peak at the sixth term. Retirement risk increases, while general election risk decreases, as the original authors pointed out. Primary hazard is almost time invariant.

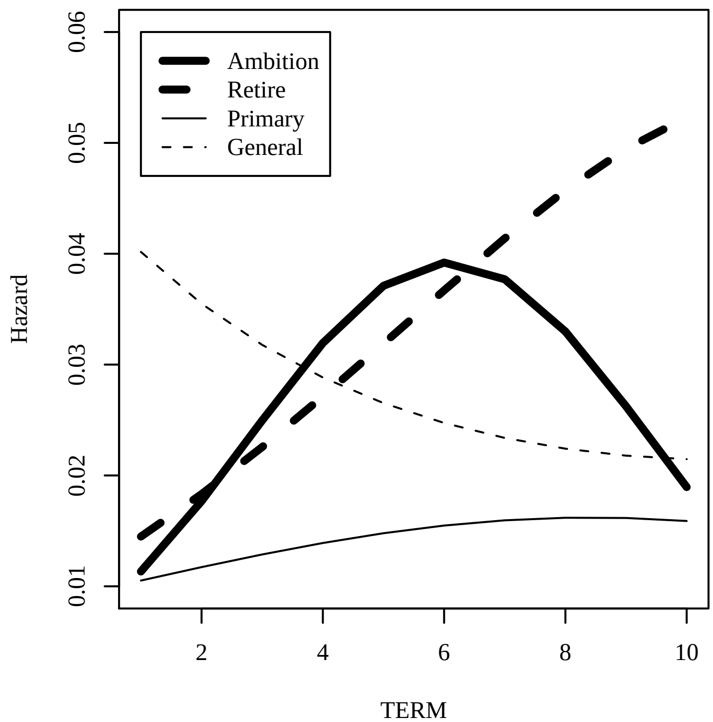

Figure 2: Time Dependence of Baseline Hazards by Risk

CONCLUSION

Competing risks model of survival analysis studies whether, when and why (or how) an event happens on a subject. Sometimes, these risks are dependent on each other. The systematically dependent competing risks model of survival analysis I proposed in this paper enables us to estimate more than two risks, which is almost impossible for currently popular frailty model. My model also distinguishes covariates' direct effects from their indirect ones and takes into consideration non-random censoring and nested risks structure.

This paper applies this model to the U.S. Congressional career path data and reveals that lawmakers strategically retire so as not to incur electoral defeat and those who are more likely to lose general election tend to fail in primary in the first place. In another paper (Fukumoto, 2005), I studies the Japanese case and show the same strategic retirement and time invariant retirement hazard. In the field of political science, there are many examples for which dependent competing risks model is useful. To name only a few, cabinet resolution, war or peace duration, and survival of administrative organization are promising applications.

It is desirable and (probably) possible, but not yet done, to combine my systematically dependent risks model and stochastically one such as frailty model. They are not exclusive. But these are future agendas.

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